From all the exercises choose 4. Solve and explain briefly your reasoning. If something is not clear, make an assumption and state it, then solve accordingly. The exam is 3h. You may hand it late, but you will receive a penalty. You will get credit for partially solved problems.

Name: ____________________

Exercises

1. As shown in figure 1, a point Q is observed in a known (i.e. intrinsic parameters are calibrated) perspective camera with image plane Π_1. Then you translate the camera parallel to the image plane with a known translation to a new image plane Π_2 and observe it again.

(1.1) Draw Q'_1 and Q'_2 on the figure. Is it possible to find the depth of the 3D point Q in this scenario? Briefly explain why.

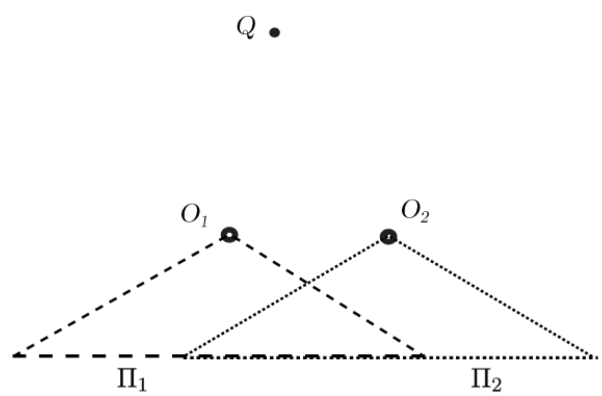


Figure 1: 3D point reconstruction.

2. Let $\mathbf{X_1}$, $\mathbf{X_2}$, and $\mathbf{X_3}$ be 3D points with homogeneous coordinates

$$\mathbf{X_1} = \begin{bmatrix}1\\2\\3\\1\end{bmatrix}, \quad \mathbf{X_2} = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \quad \mathbf{X_3} = \begin{bmatrix}2\\3\\0\\1\end{bmatrix}, \tag{1}$$

and a camera with camera matrix

$$P = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\end{bmatrix}. \tag{2}$$

(2.1) Compute the projections of the 3D points in P.

(2.2) What is the interpretation of the projection of X_3?

(2.3) Compute the camera center (position) of the camera.

3. Suppose that a camera has got the inner parameters

$$K = \begin{bmatrix}f & 0 & x_0\\0 & f & y_0\\0 & 0 & 1\end{bmatrix}. \tag{3}$$

(3.1) Verify that the inverse of K is

$$K^{-1} = \begin{bmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

(3.2) and that this matrix can be factorized into

$$K^{-1} = \underbrace{\begin{bmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{=A} \underbrace{\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{=B}. \tag{5}$$

(3.3) What is the geometric interpretation of the transformations A and B?

4. When normalizing the image points of a camera with known inner parameters we apply the transformation K^{-1}?

(4.1) What is the interpretation of this operation?

(4.2) Where does the principal point (x_0, y_0) end up?

(4.3) And where does a point with distance f to the principal point end up?

5. Suppose that for a camera with resolution 640×480 pixels we have the inner parameters

$$K = \begin{bmatrix} 320 & 0 & 320 \\ 0 & 320 & 240 \\ 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

(5.1) Normalize the points (0, 240), (640, 240).

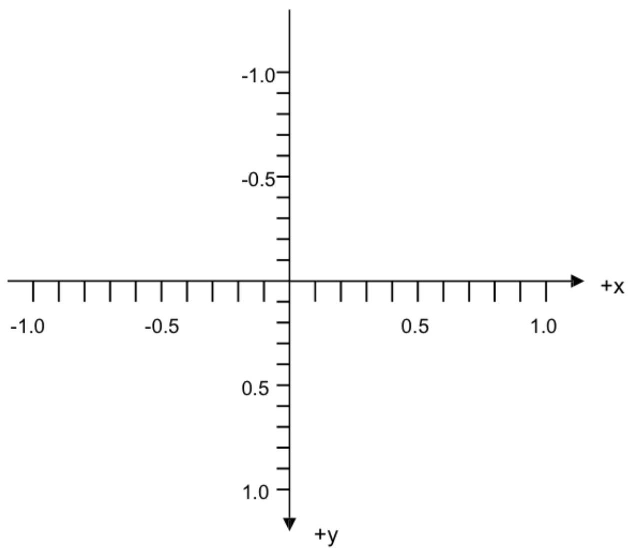
(5.2) What is the angle between the viewing rays projecting to these points?

6. Given an Essential matrix

$$\mathbf{E} = \begin{bmatrix} 0 & 0 & 0.5 \\ 0 & 0 & -1 \\ -0.5 & 1 & 0 \end{bmatrix}, \tag{7}$$

that relates the image of a point in one camera to its image in the other camera. Assume that a point is observed at image location $(0.3, 0.1)$ in the second image, where these are "normalized" image coordinates (such that effective focal length is equal to 1).

(6.1) Accurately draw the corresponding epipolar line in the first image, on the graph below.


-1.0
-0.5
+x
-1.0
-0.5
0.5
1.0
0.5
1.0
+y