Optical Metrology

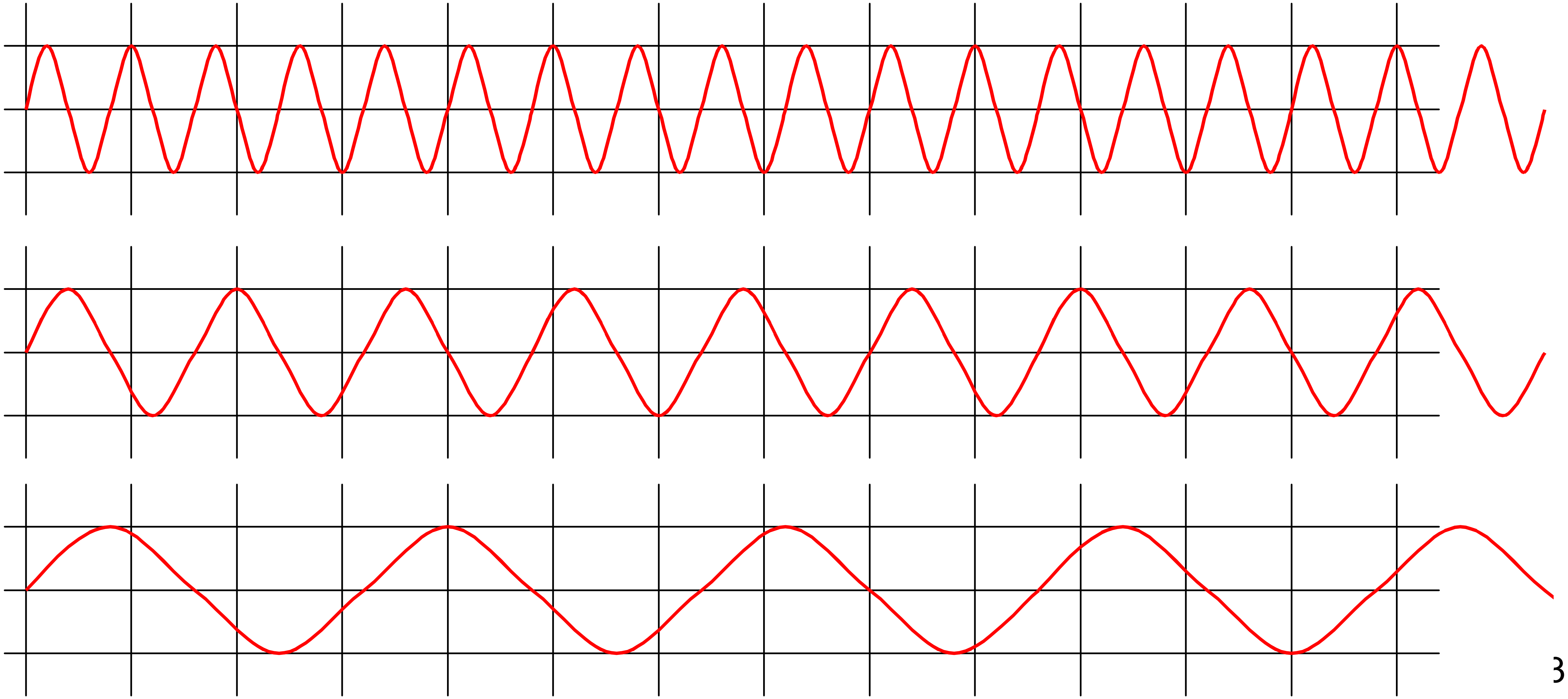
Lecture 6: Interferometry

Outline

- Introduction
- General Description
- Coherence
- Interference between 2 plane waves
  - Laser Doppler velocimetry
- Interference between spherical waves
- Interferometry
  - Wavefront Division
  - Amplitude Division
- Heterodyne Interferometry

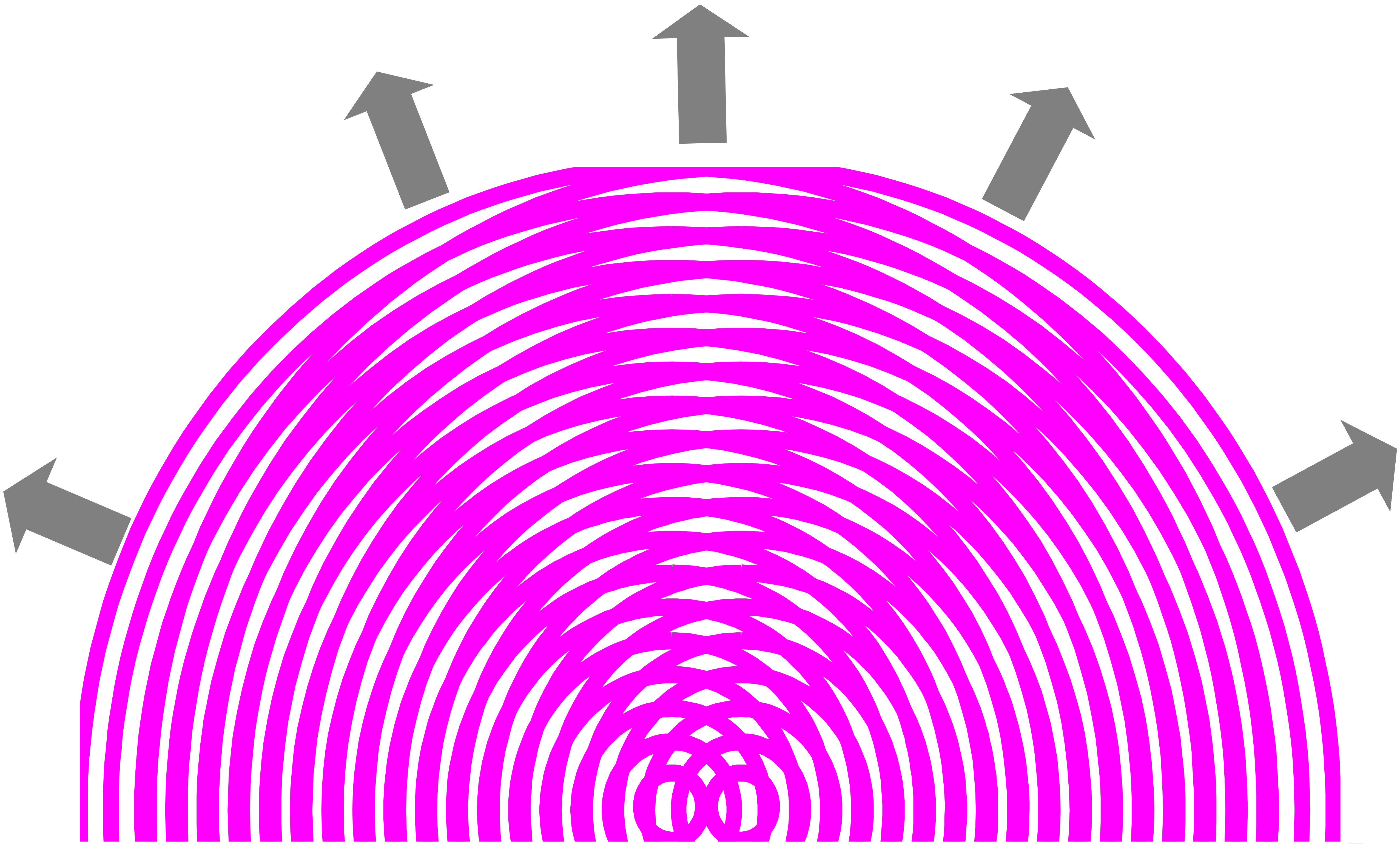
Light as Waves

- Waves have a wavelength
- Waves have a frequency



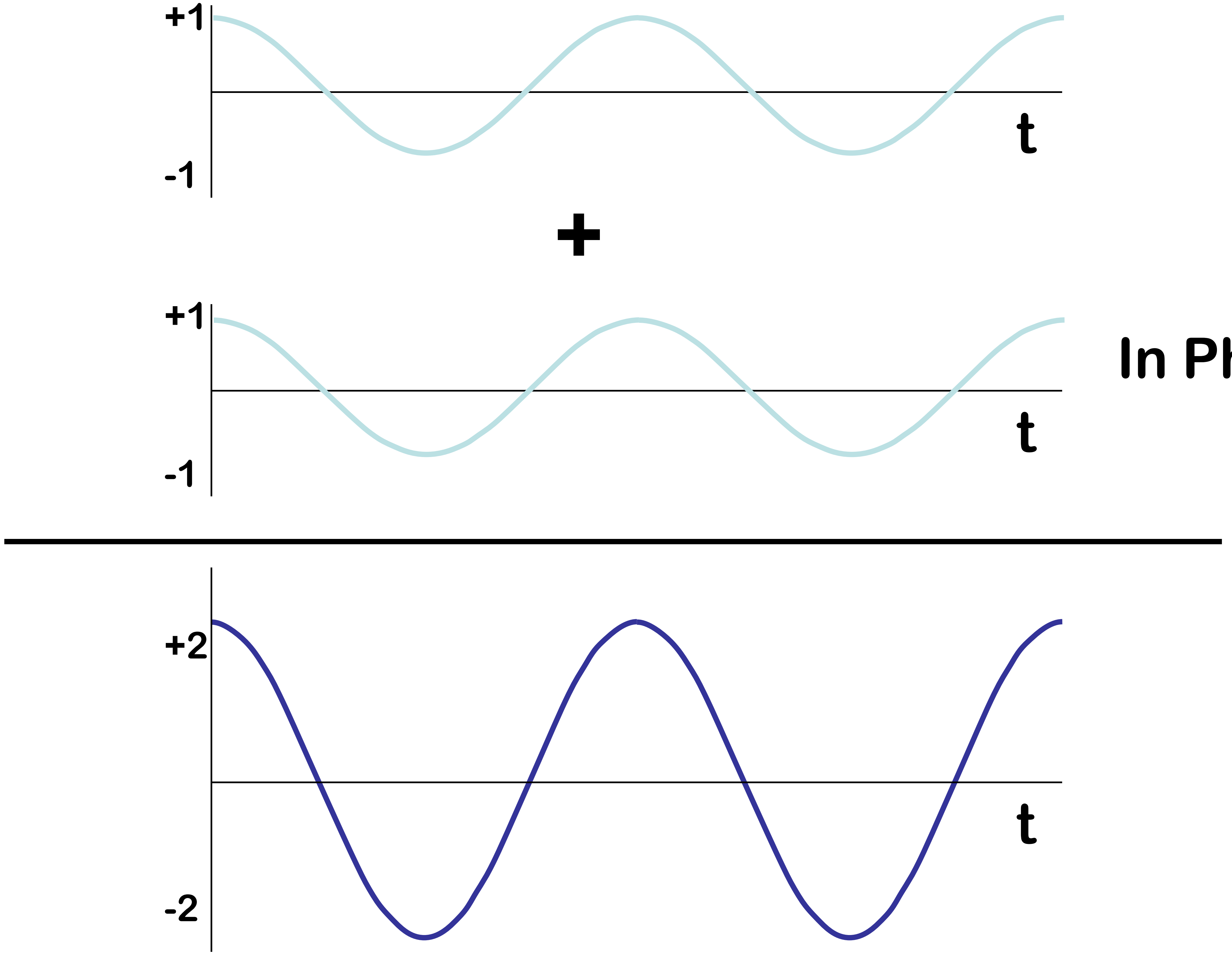
Interference in water waves…

Overlapping Semicircles



Superposition

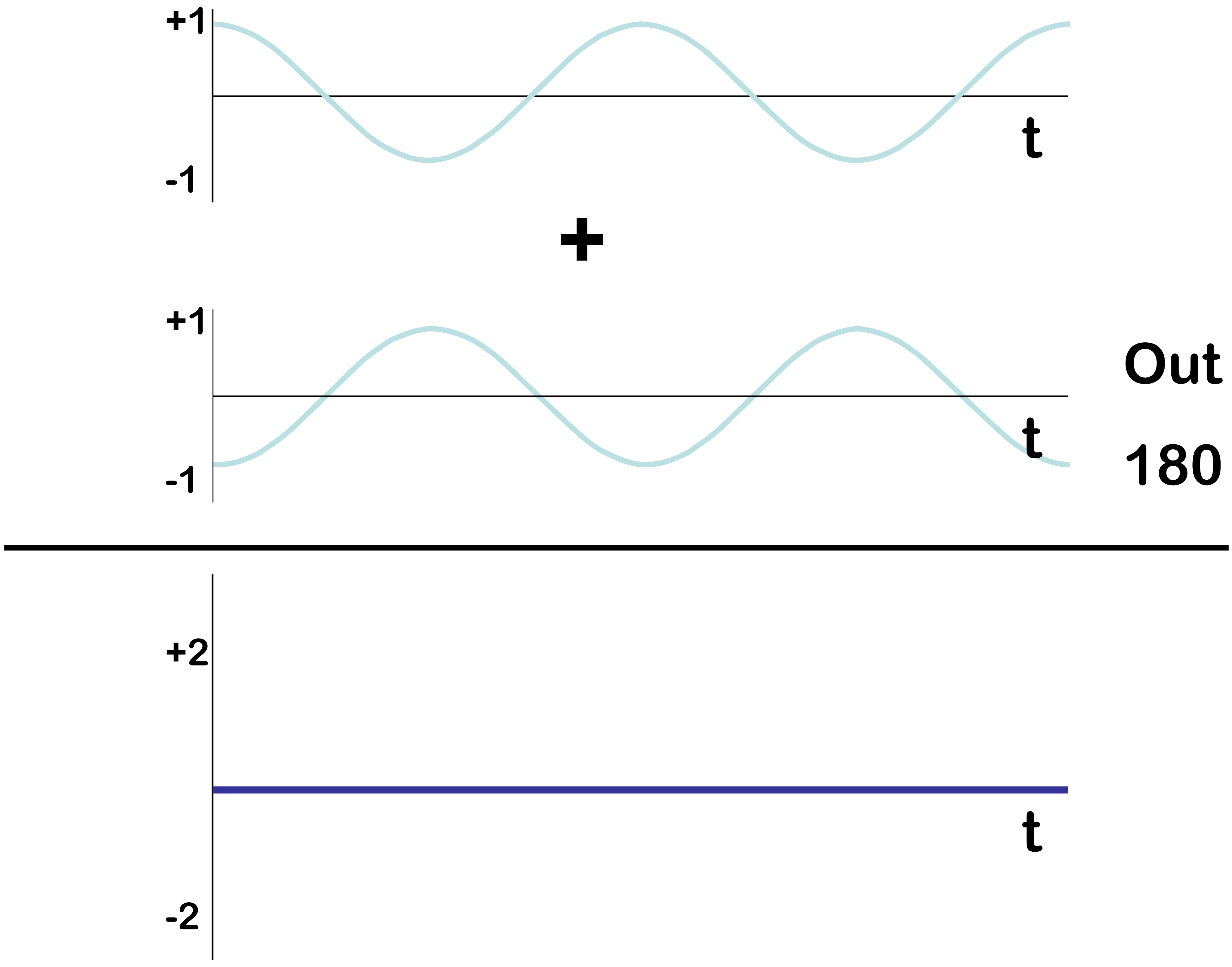
Constructive Interference



In Phase

Superposition

Destructive Interference

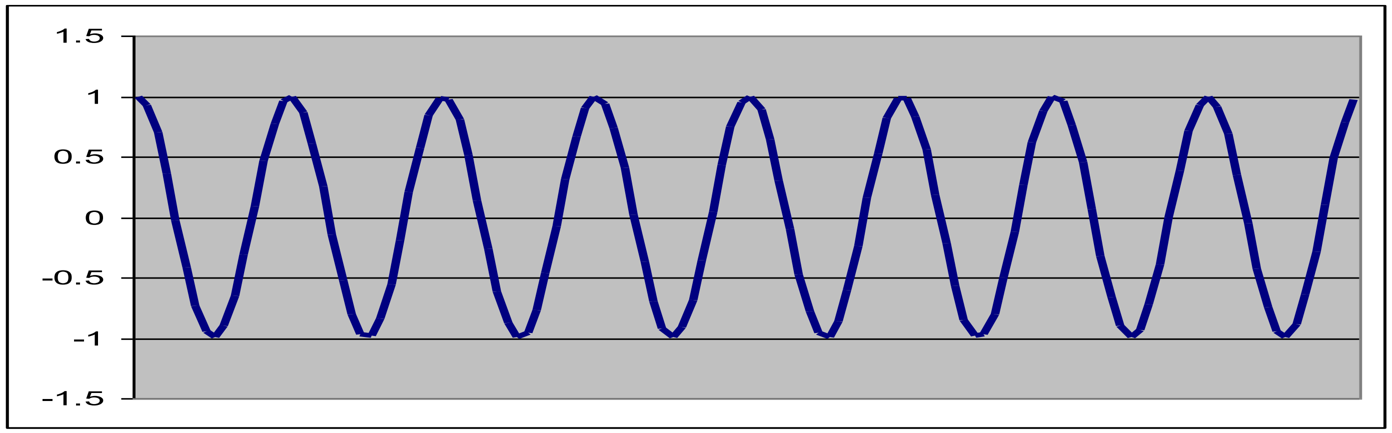


Out of Phase

180 degrees

Superposition

Different f



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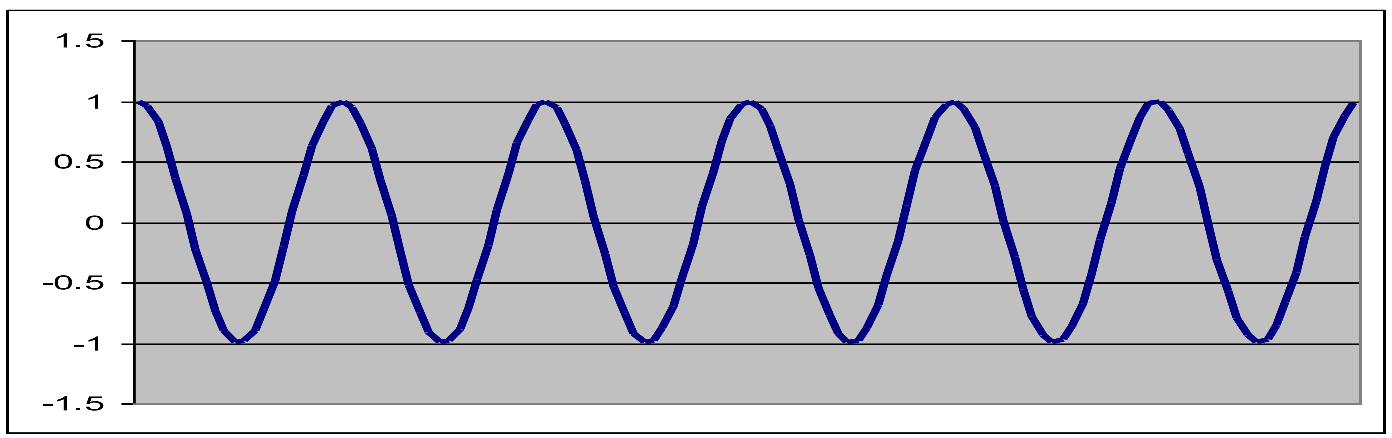
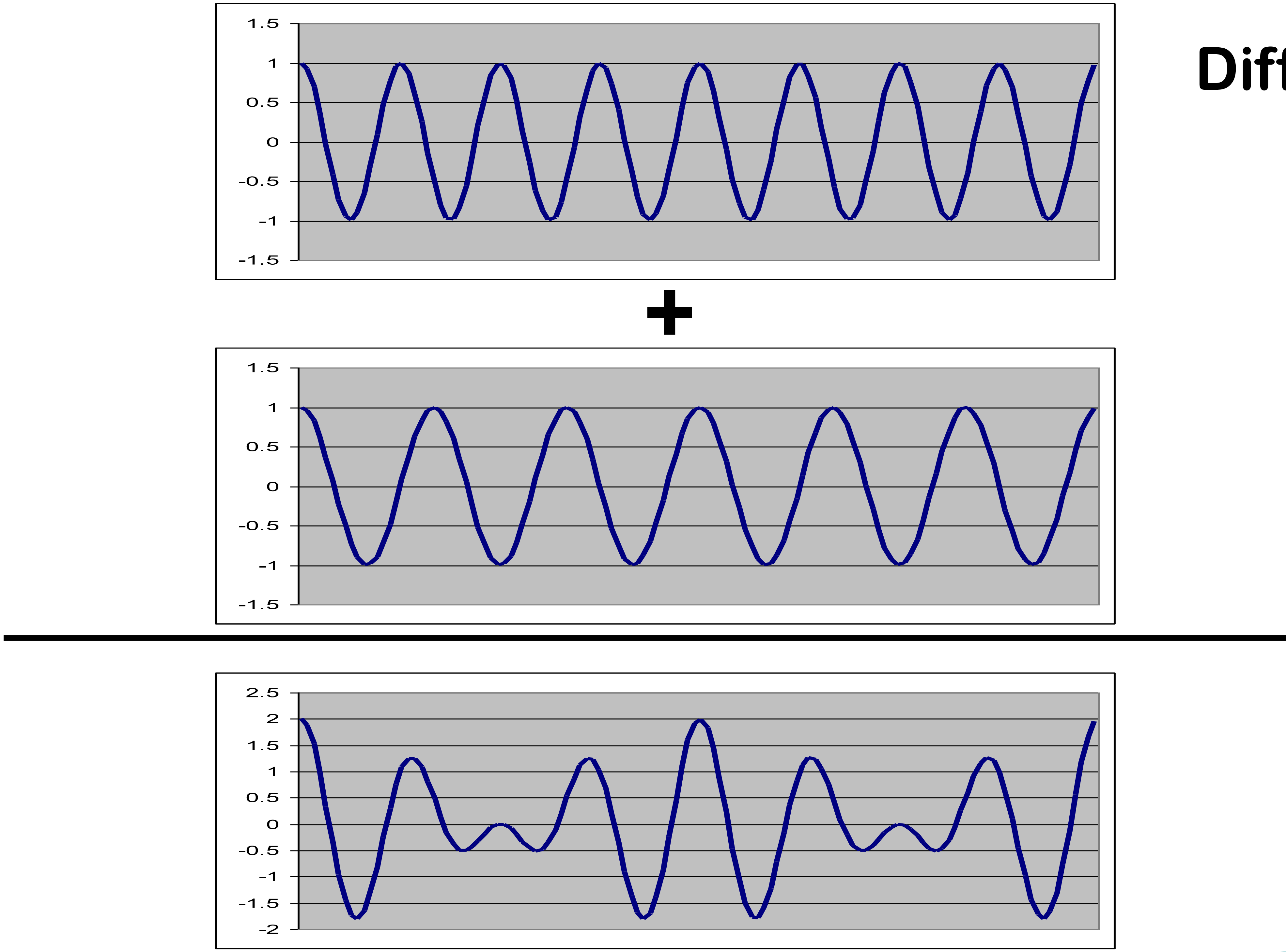


1) Constructive **2) Destructive** **3) Neither**

Superposition



Different f

1) Constructive **2) Destructive** **3) Neither**

Interference Requirements

- Need two (or more) waves
- Must have same frequency
- Must be coherent (i.e. waves must have definite phase relation)

General Description

- Interference can occur when two or more waves overlap each other in space. Assume that two waves described by $u_1 = U_1 e^{i\phi_1}$ and $u_2 = U_2 e^{i\phi_2}$ overlap
- The electromagnetic wave theory tells us that the resulting field simply becomes the sum $u = u_1 + u_2$
- The observable quantity is intensity (irradiance) I which is

$$I = |u|^2 = |u_1 + u_2|^2 = U_1^2 + U_2^2 + 2U_1U_2\cos(\phi_1 - \phi_2)$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$$

- Where $e^{i\phi} = (\cos\phi + i\sin\phi)$ and $\Delta\phi = \phi_1 - \phi_2$

General Description

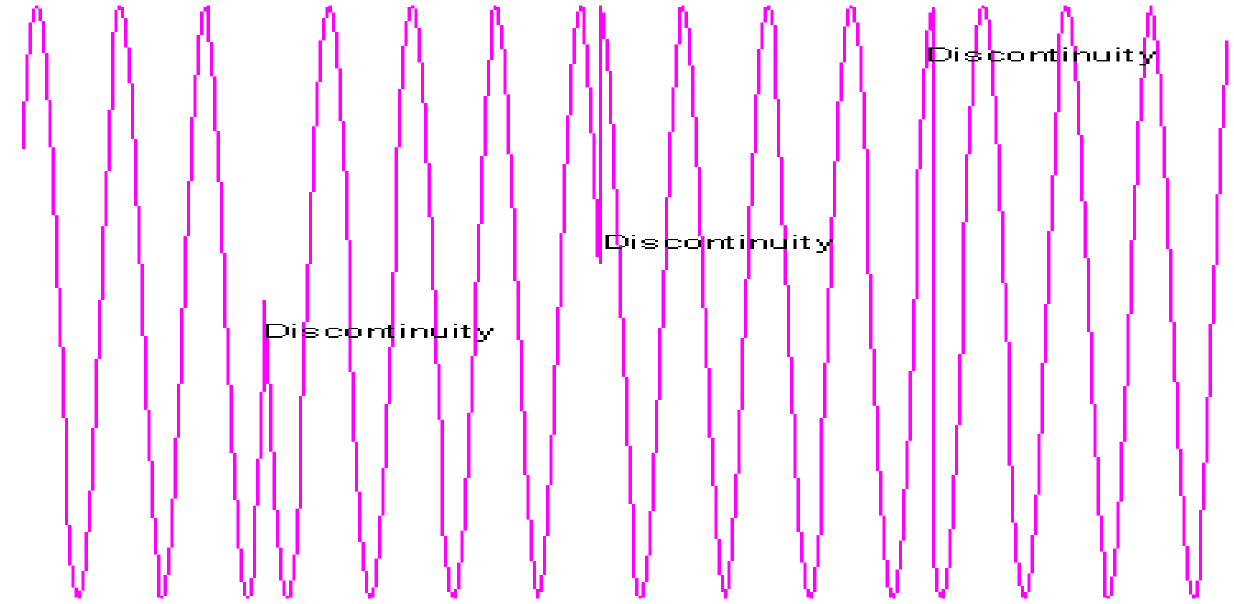
- Resulting intensity is not just ($I_1 + I_2$).
- When 2 waves interfere $2\sqrt{I_1 I_2}\cos\Delta\phi$ is called the interference term
- We also see that when $\Delta\phi = (2n+1)\pi, \quad \text{for} \quad n = 0, 1, 2, \ldots$ then $\cos\Delta\phi = -1$ and I reaches minima (cos 180°) which means destructive interference
- Similarly when $\Delta\phi = 2n\pi, \quad \text{for} \quad n = 0, 1, 2, \ldots$ then $\cos\Delta\phi = 1$ and I reaches maxima (cos 0°) constructive interference
- When 2 waves have equal intensity $I_1 = I_2 = I_0$

$$I = 2I_0[1 + \cos\Delta\phi] = 4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$$

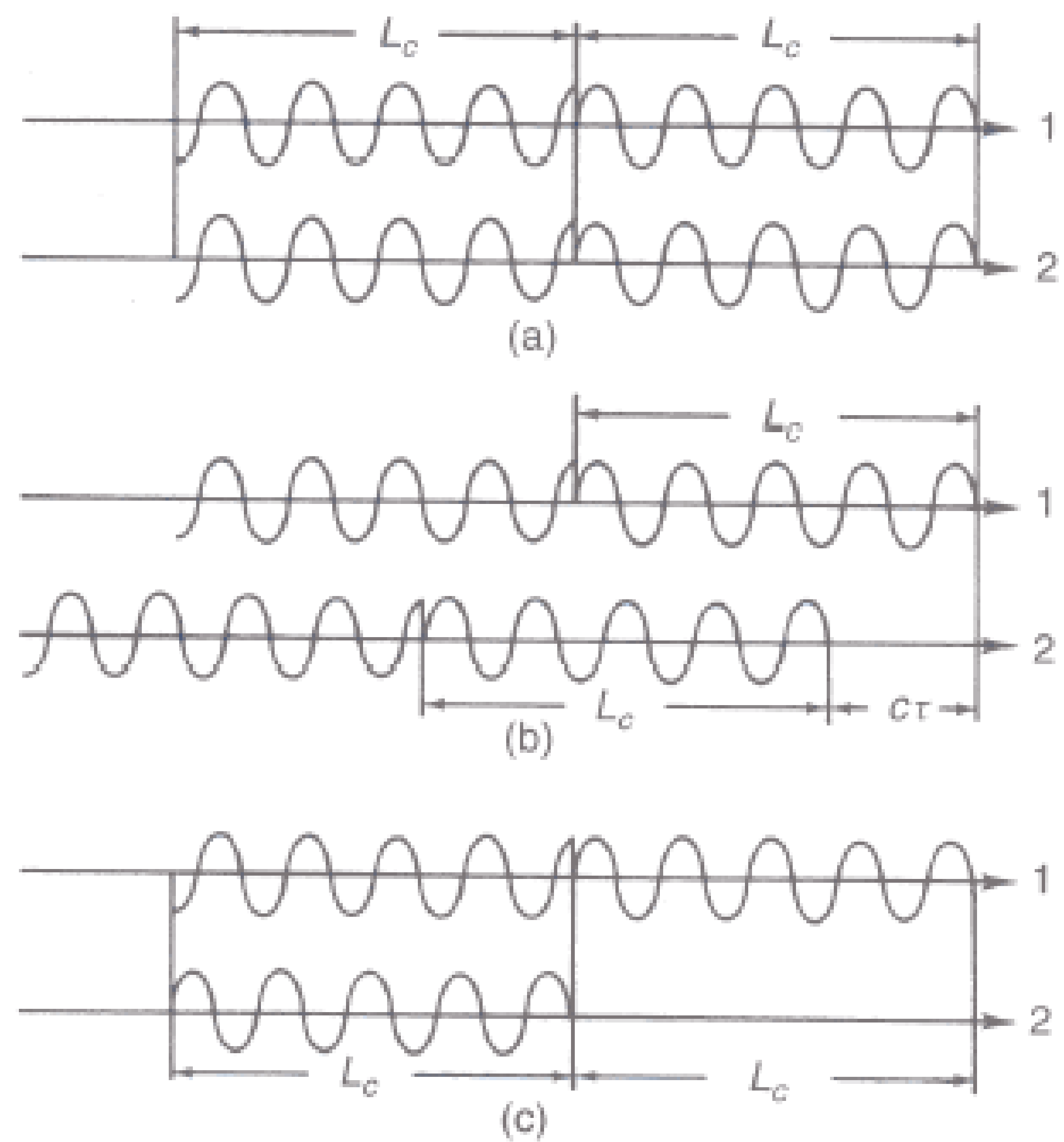
Coherence

- Detection of light is an averaging process in space and time
- We assume that ul and u2 to have the same single frequency
- Light wave with a single frequency must have an infinite length
- However sources emitting light of a single frequency do not exist

Coherence

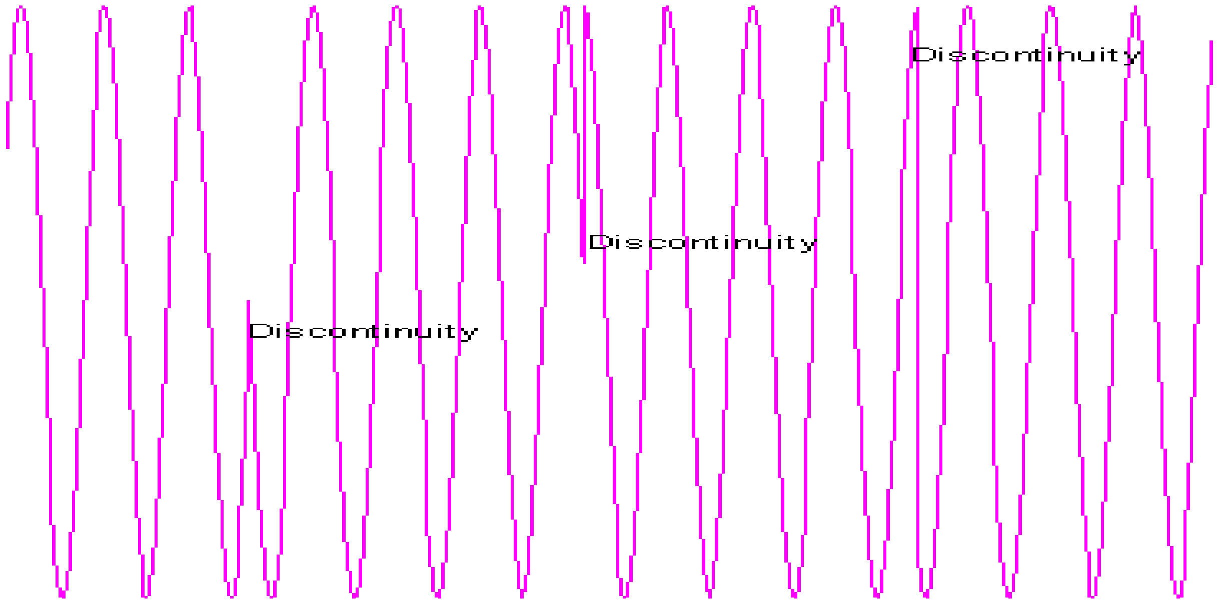


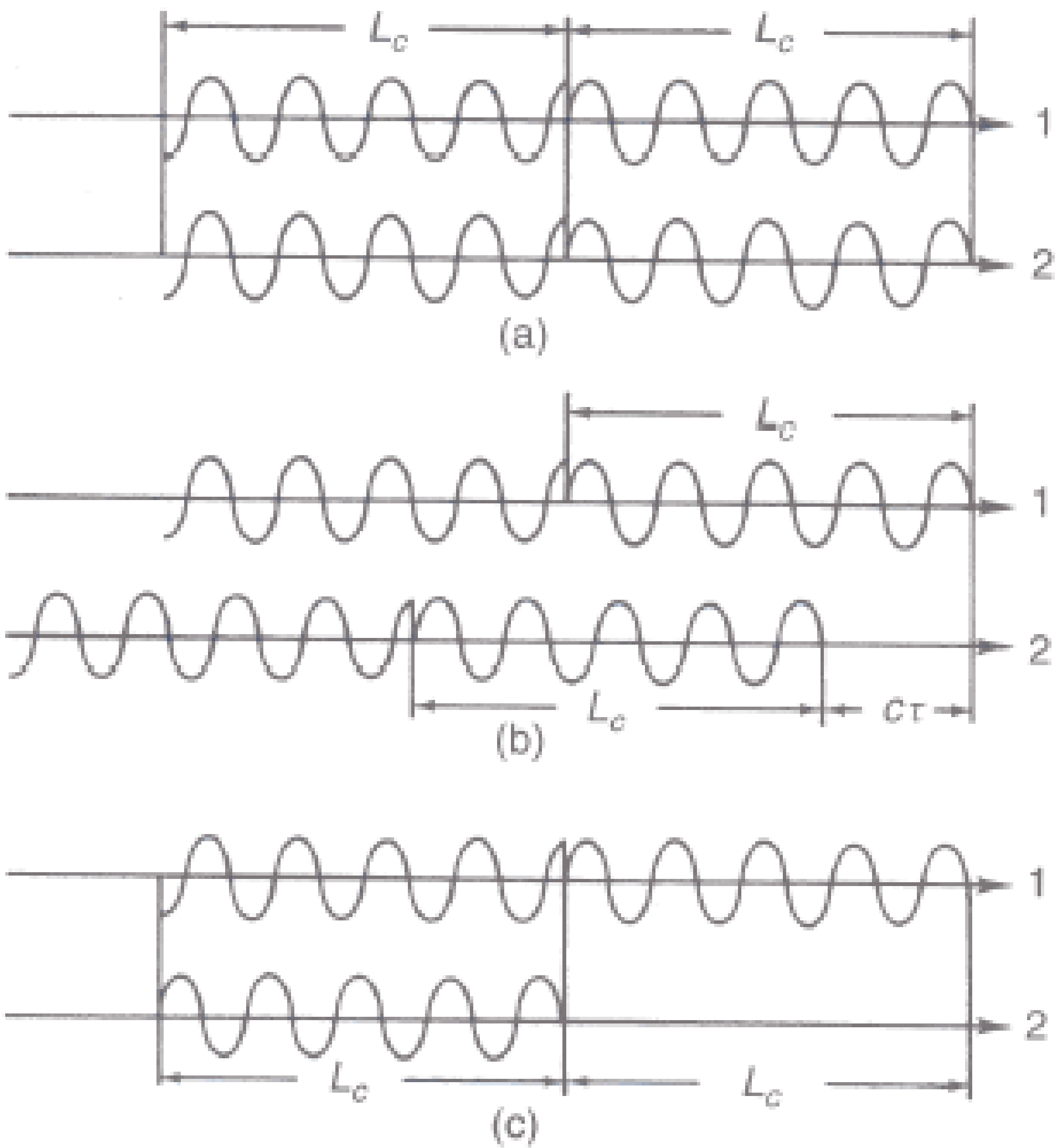
- Here we see two successive wave trains of the partial waves
- The two wave trains have equal amplitude and length Lc, with an abrupt, arbitrary phase difference
- a) shows the situation when the two waves have traveled equal paths. We see that although the phase of the original wave fluctuates randomly, the phase difference remains constant in time $= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi$



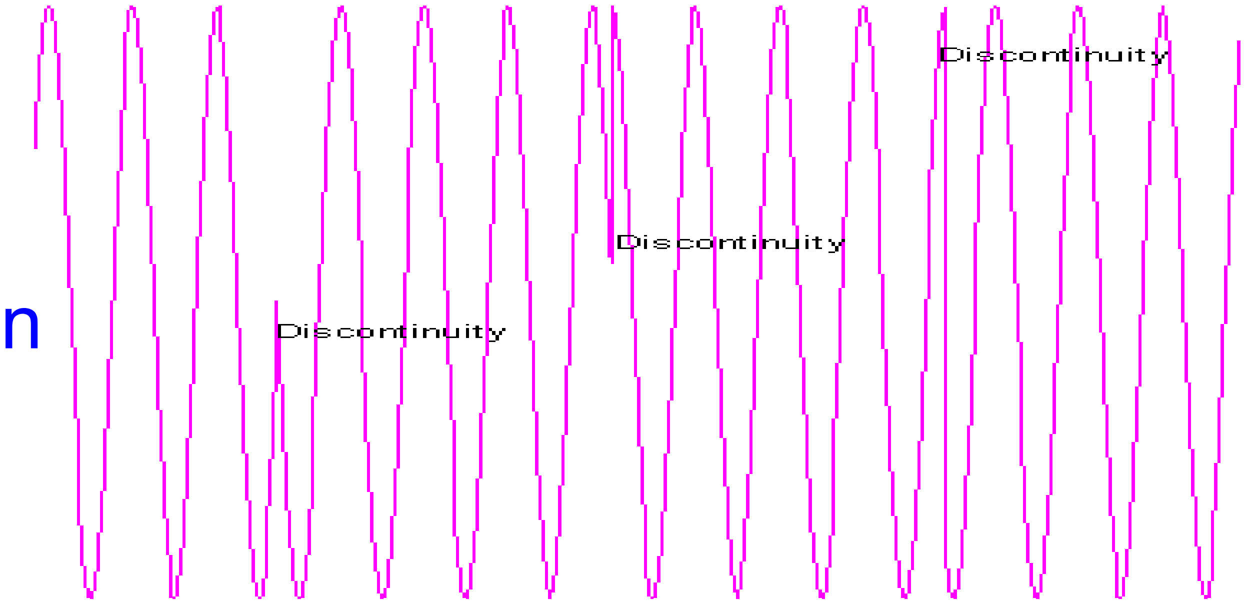
Coherence

- In c) wave 2 has traveled Lc longer than wave 1. The head of the wave trains in wave 2 coincide with tail of the corresponding wave trains in partial wave 1.
- Now the phase difference fluctuates randomly as the successive wave trains pass by
- Here cos $\Delta\phi$ varies randomly between +1 and -1 and for multiple trains it becomes 0 (no interference) $I = I_1 + I_2$

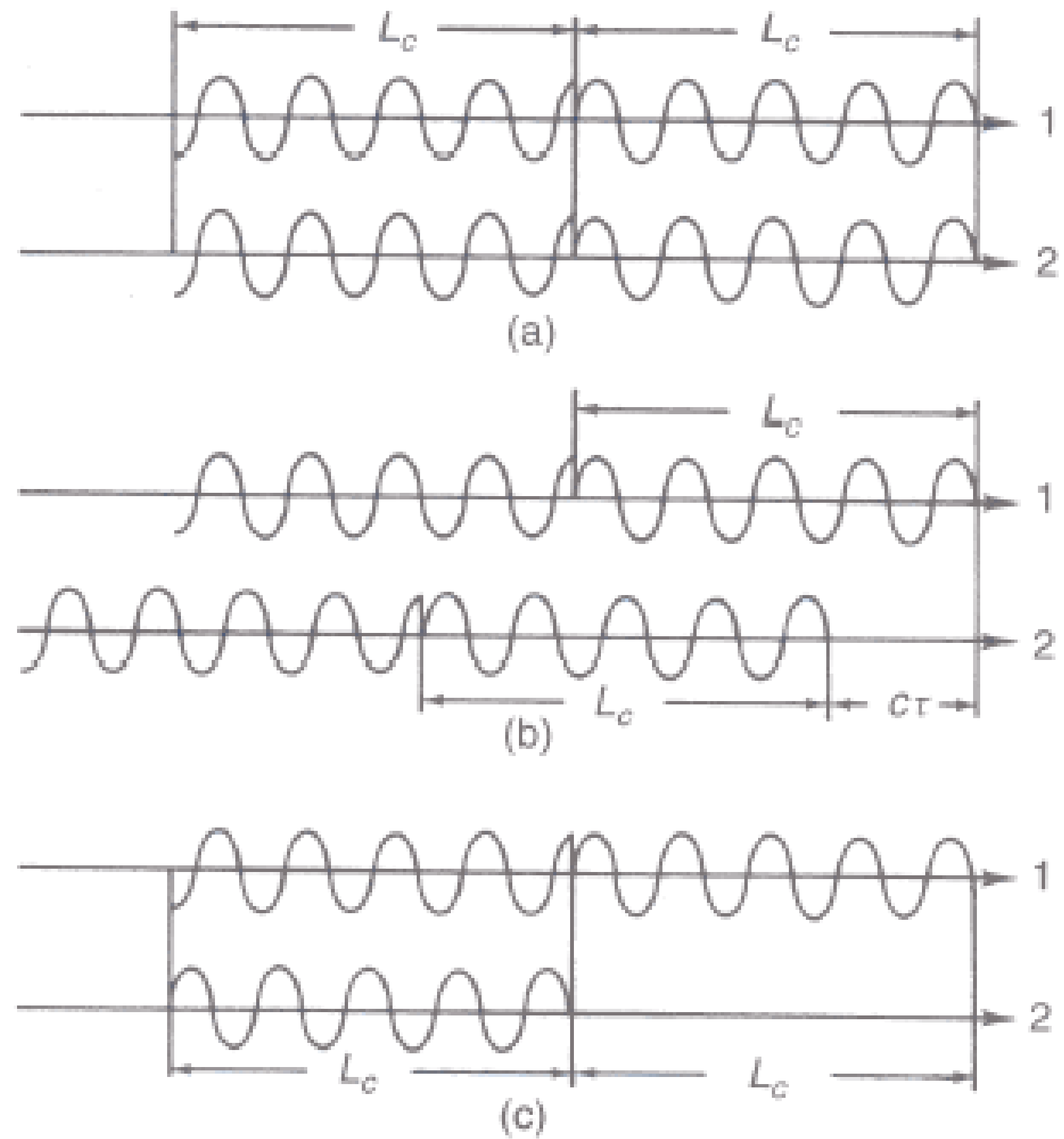




Coherence



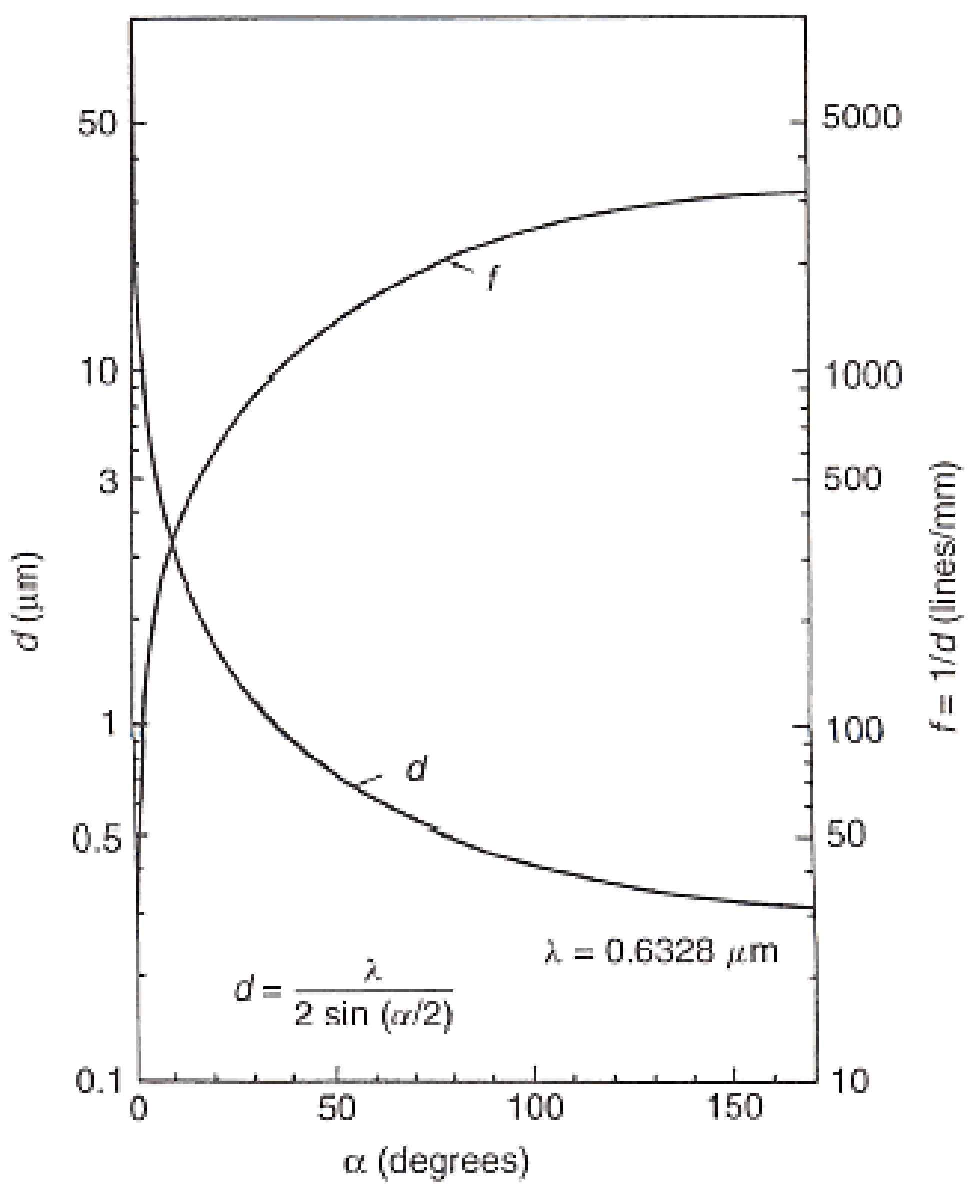
- In b) wave 2 has traveled l longer than wave 1 where $0<l<L_c$.
- For many wavetrains the phase difference varies in time proportional to $\tau_c - \tau$ where $\tau_c = L_c/c$.
- we still can observe an interference pattern but with a reduced contrast
- L_c is the coherence length and is the coherence time τ_c
- For white light, the coherence length is 1 micron

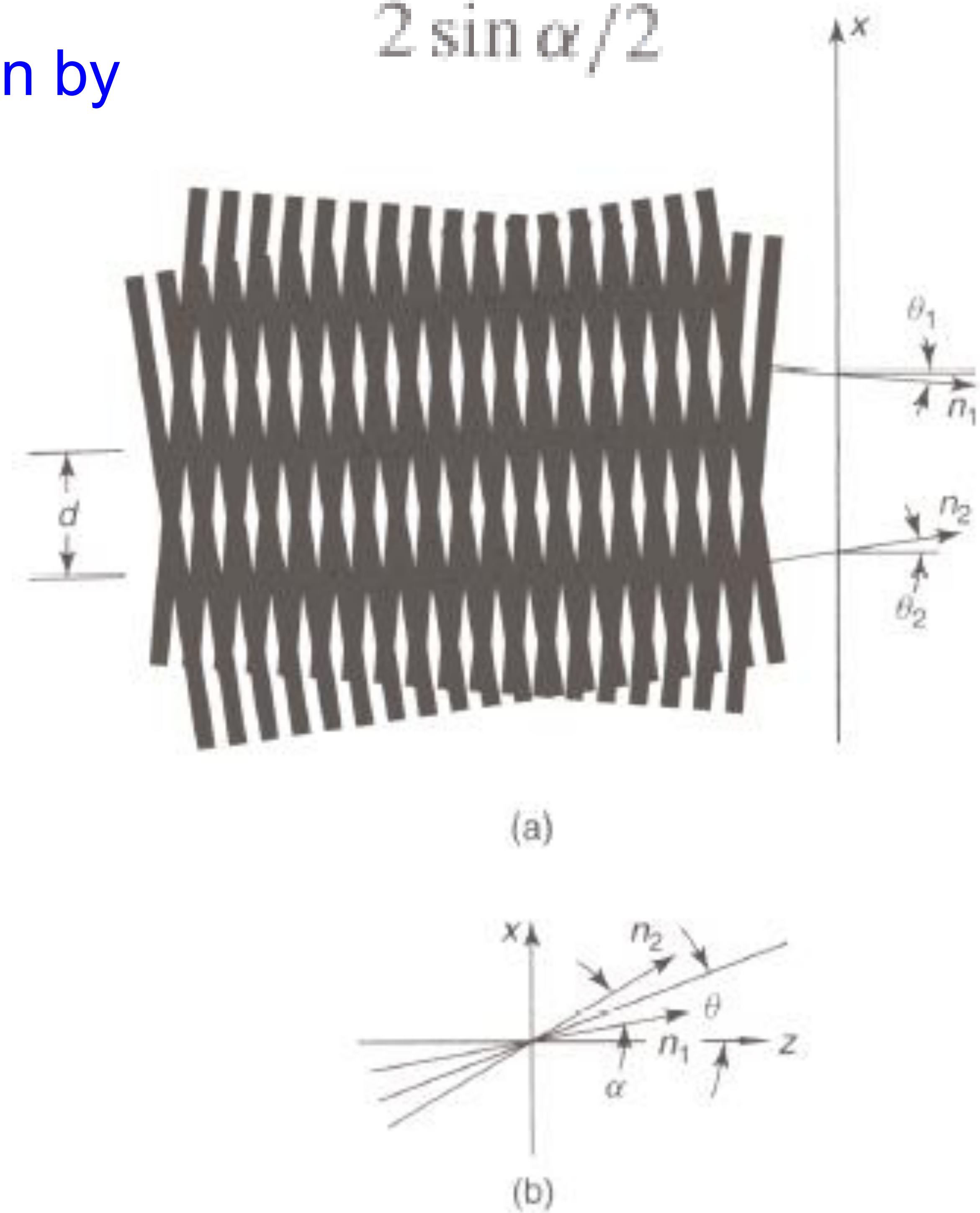


Plane Wave Interference

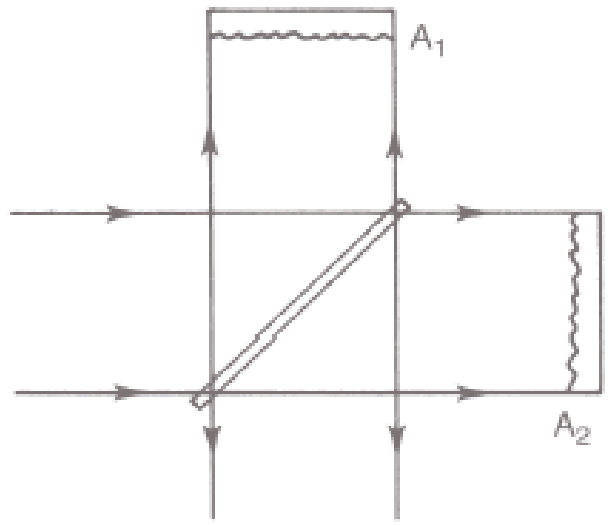
- When two plane wave interfere the resultant fringe spacing is given by

$$d = \frac{\lambda}{2 \sin \alpha / 2}$$





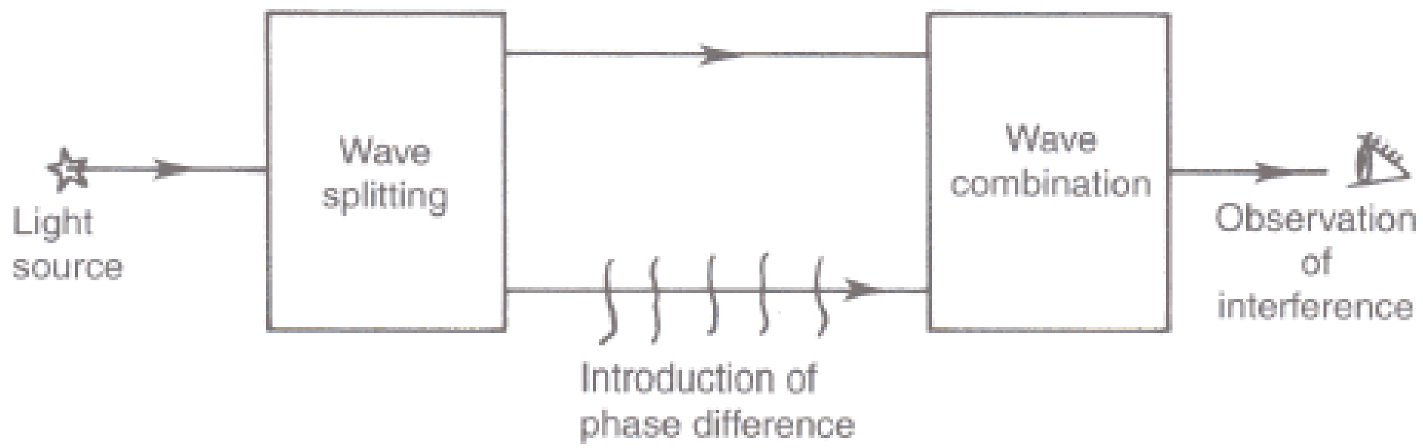
Interference between other Waves



- By measuring the distance between interference fringes over selected planes in space, quantities such as the angle and distance can be found.
- One further step would be to apply for a wave reflected from a rough surface
- By observing the interference - can determine the surface topography
- For smoother surfaces, however, such as optical components (lenses, mirrors, etc.) where tolerances of the order of fractions of a wavelength are to be measured, that kind of interferometry is quite common.

Interferometry

- Light waves interfere only if they are from the same source (why???)



- Most interferometers have the following elements
  - light source
  - element for splitting the light into two (or more) partial waves
  - different propagation paths where the partial waves undergo different phase contributions
  - element for superposing the partial waves
  - detector for observation of the interference

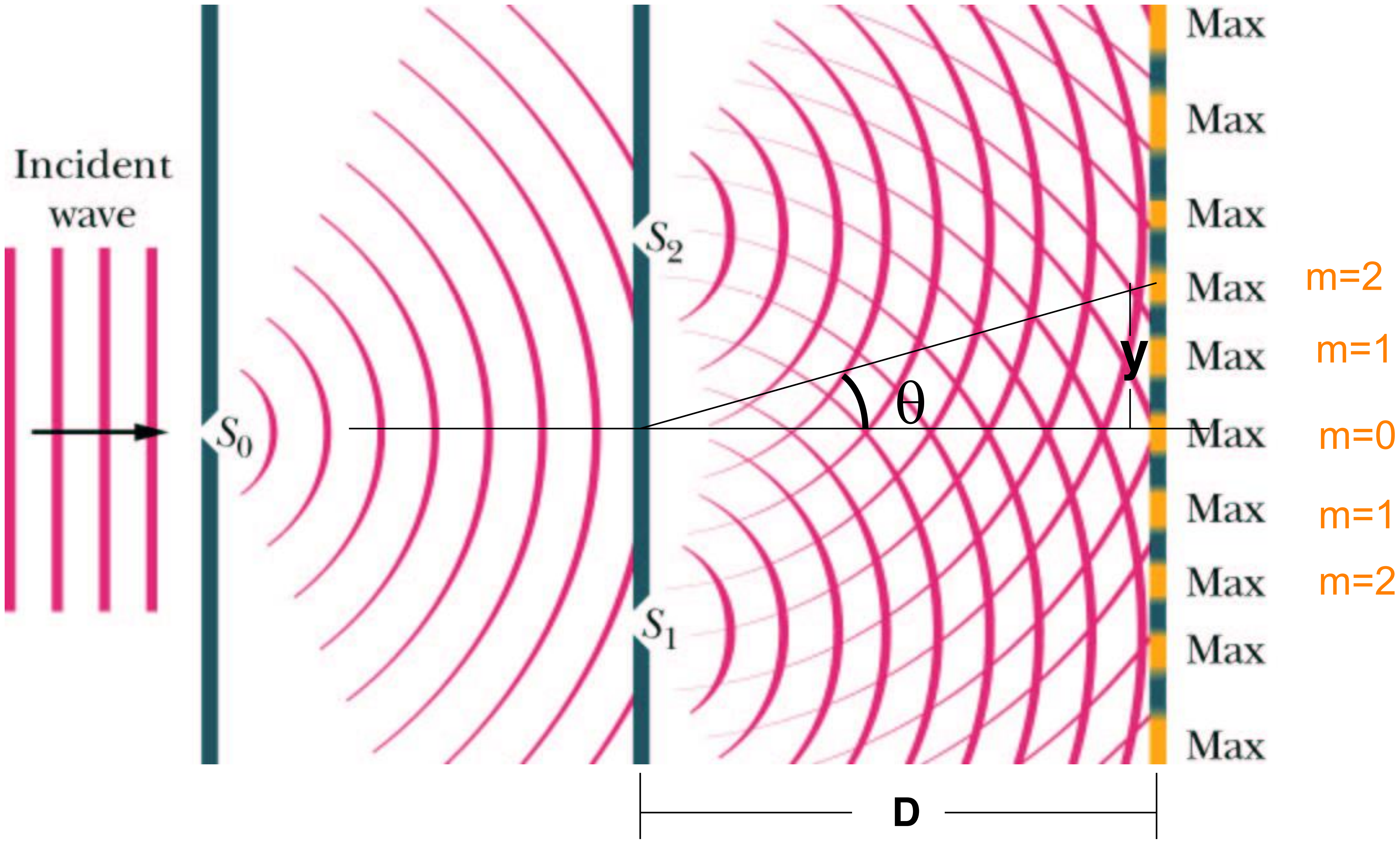
Interferometry

- Depending on how the light is split, interferometers are commonly classified
  - Wavefront division interferometers
  - Amplitude division interferometers

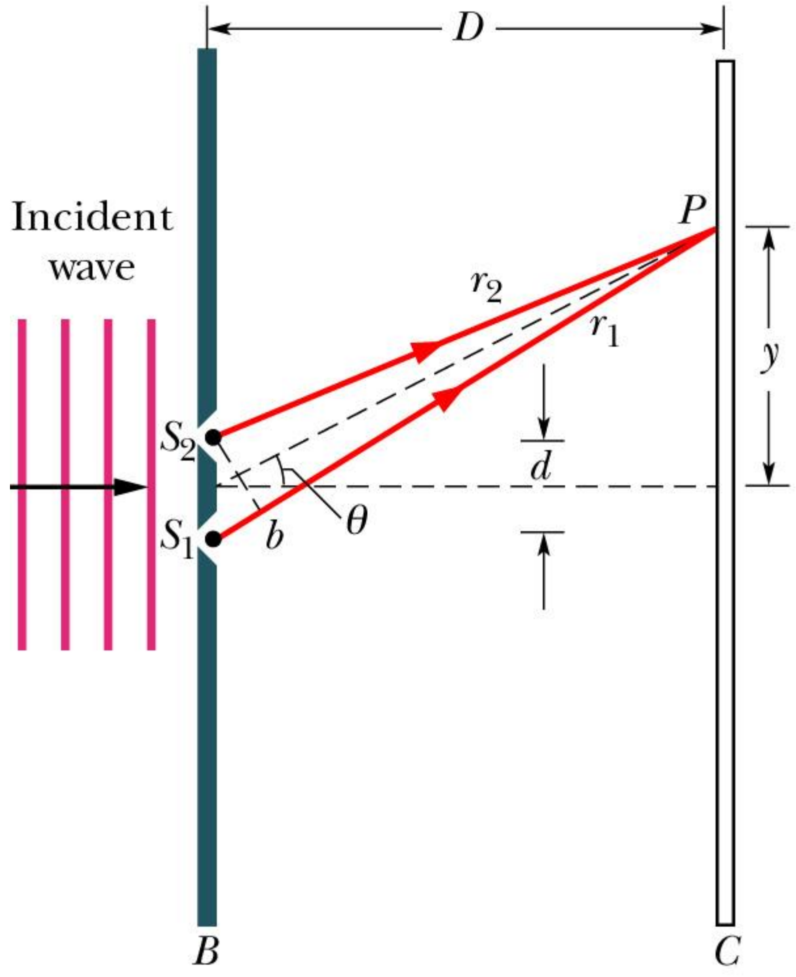
Wavefront Division

- Example of a wavefront dividing interferometer, (Thomas Young)
- The incident wavefront is divided by passing through two small holes at S_l and S_2 in a screen 1.
- The emerging spherical wavefronts from S_l and S_2 will interfere, and the pattern is observed on screen 2.
- The path length differences of the light reaching an arbitrary point x on S_2 is found from Figure
- When the distance D between screens is much greater than the distance d between S_1 and S_2, we have a good approximation

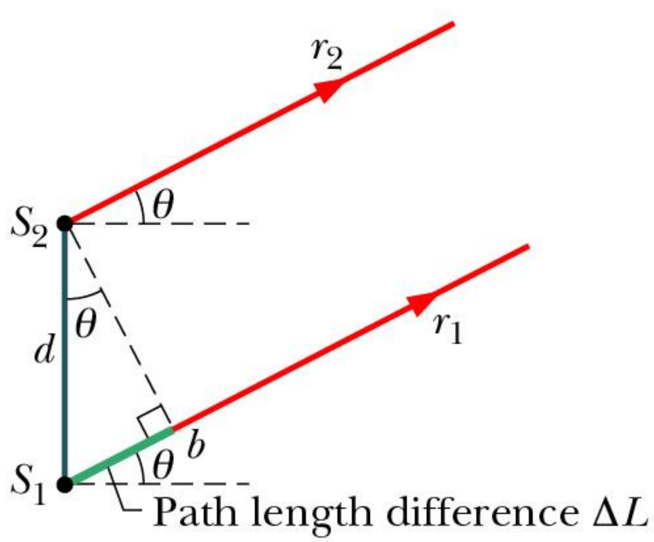
Wavefront Division



Wavefront Division



(a)



$= d\sin\theta$

(b)

$$d\sin\theta = m\lambda \quad \mathrm{m} = 0,1,2,3\ldots \quad \text{Maximum}$$

$$\tan\theta = \frac{y_m}{D} \quad \text{or} \quad y_m = D\tan\theta \approx D\sin\theta$$

$$y_m = \frac{m\lambda D}{d}$$

Maxima

m	y_m +/-
0	0
1	Dλ/d
2	2Dλ/d
3	3Dλ/d

Minima

m	y_m +/-
0	Dλ/2d
1	3Dλ/2d
2	5Dλ/2d
3	7Dλ/2d

$$d\sin\theta = (m + \frac{1}{2})\lambda \quad \mathrm{m} = 0,1,2,3\ldots \quad \text{Minimum}$$

$$y_m = \frac{(m + 1/2)\lambda D}{d}$$

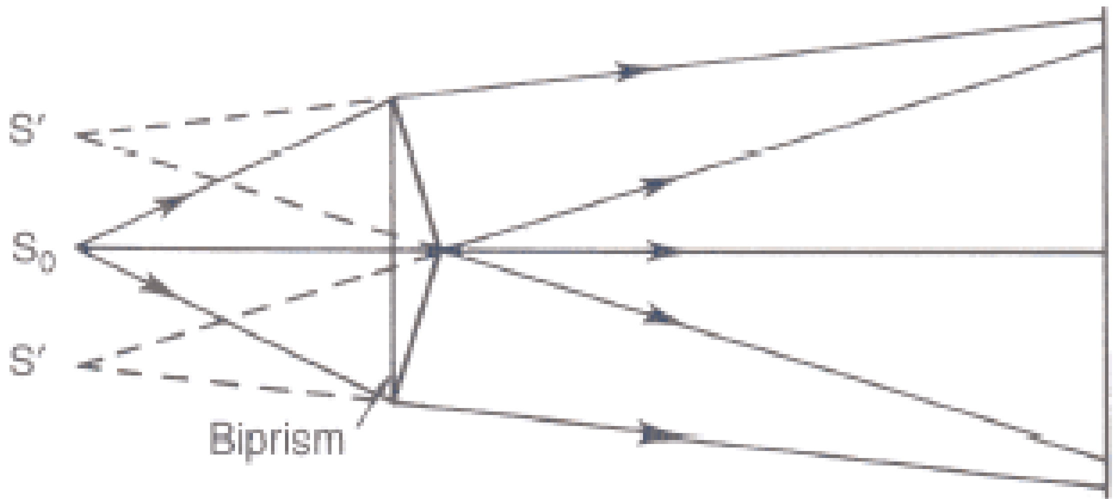
Wavefront Division

13E Suppose that Young's experiment is performed with blue-green light of 500 nm. The slits are 1.2mm apart, and the viewing screen is 5.4 m from the slits. How far apart the bright fringes?

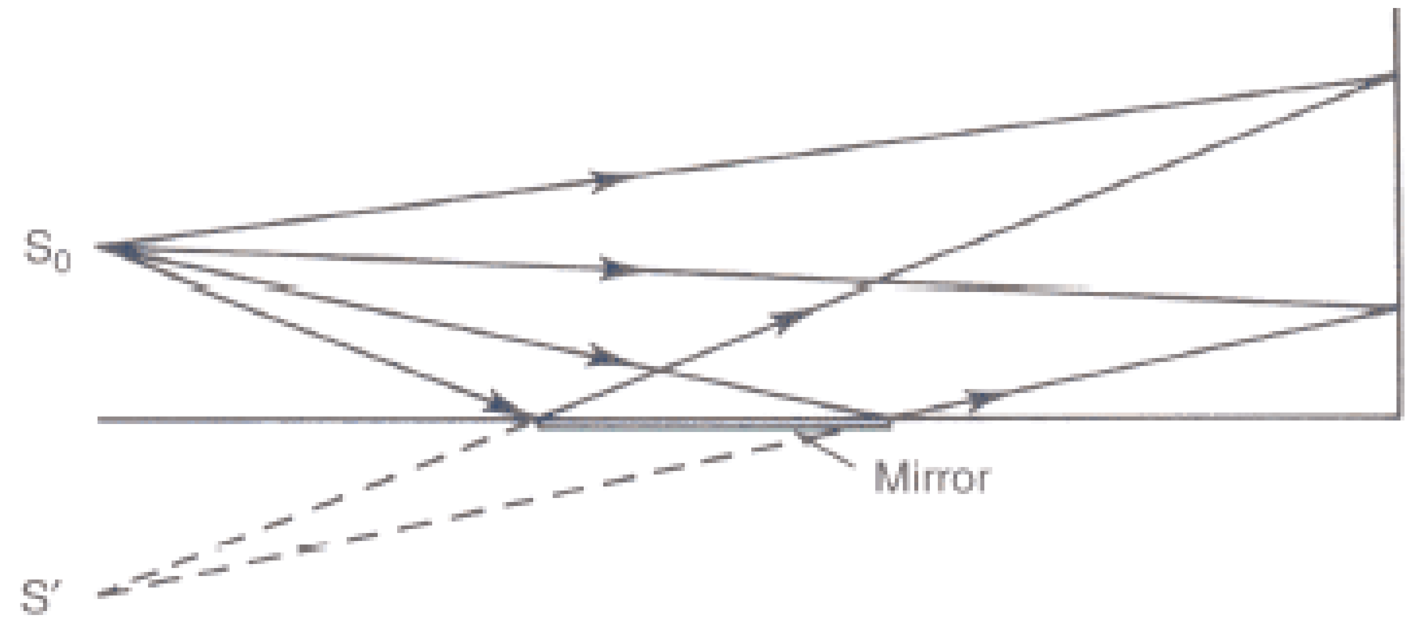
From the table on the previous slide we see that the separation between bright fringes is $D\lambda / d$

$$D\lambda / d = (5.4m)(500 \times 10^{-9} m)/0.0012m$$
$$= 0.00225m = 2.25mm$$

Wavefront Division



(a)

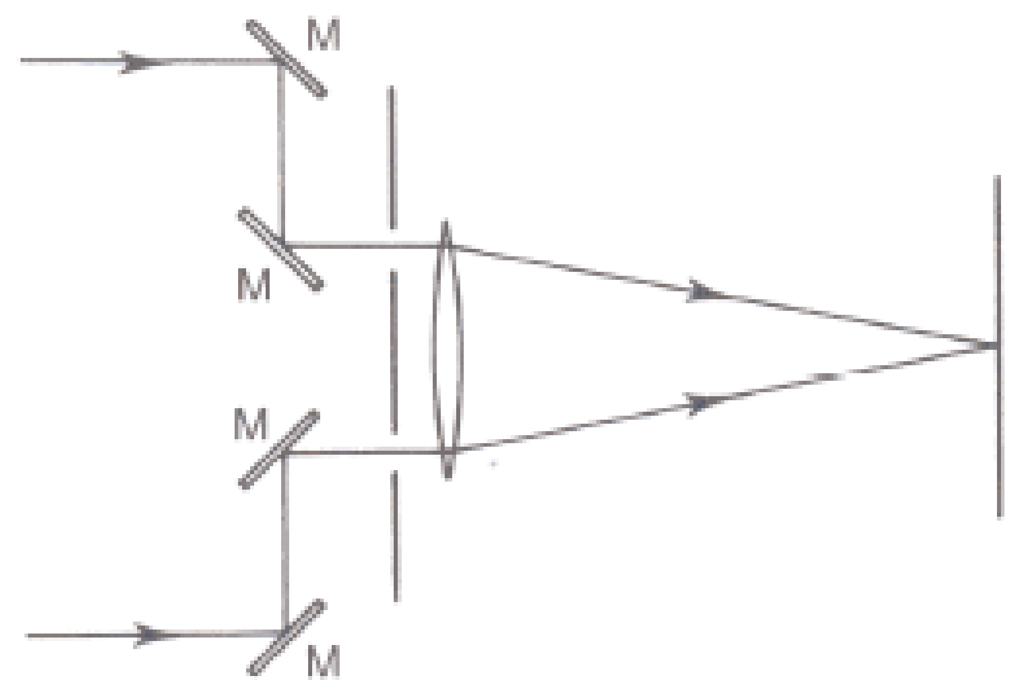


(b)

A) Fresnel Biprism

B) Lloyds Mirror

C) Michelsons Stellar Interferometer



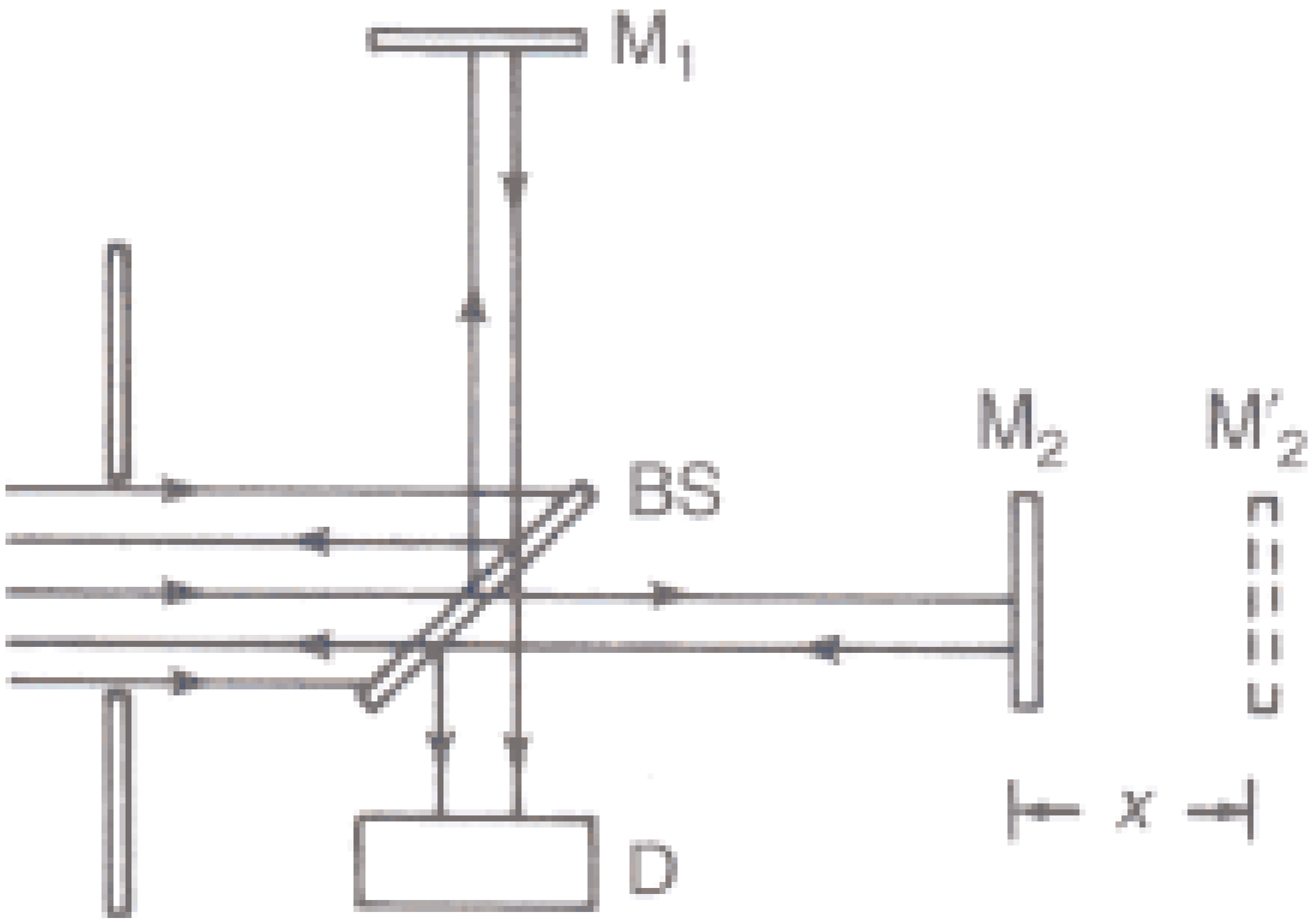
(c)

Amplitude Division

- Example of a amplitude dividing interferometer, (Michelson)
- Amplitude is divided by beamsplitter BS which partly reflects and partly transmits
- These divided light go to two mirrors M_1 and M_2 where they are reflected back.
- The reflected lights recombine to form interference on the detector D
- The path length can be varied by moving one of the mirrors or by mounting that on movable object (movement of x give path difference of 2x) and phase difference $\Delta\phi = (2\pi/\lambda)2x.$

Amplitude Division

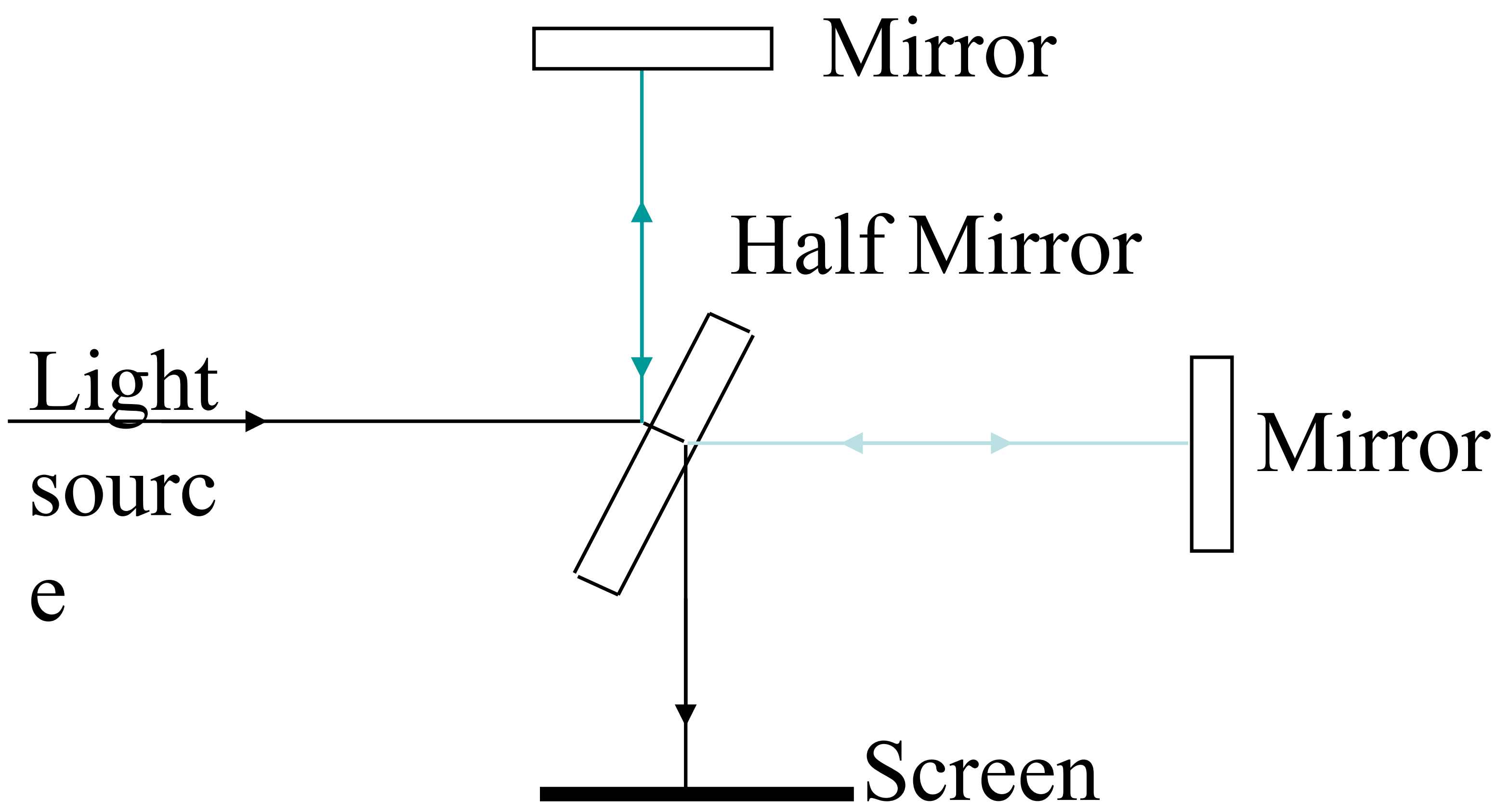
- As M_2 moves the displacement is measured by counting the number of light maxima registered by D
- By counting the number of maxima per unit time will give the velocity of the object.
- The intensity distribution is given by



$$I(x) = 2I\left(1 + \cos\frac{4\pi x}{\lambda}\right)$$

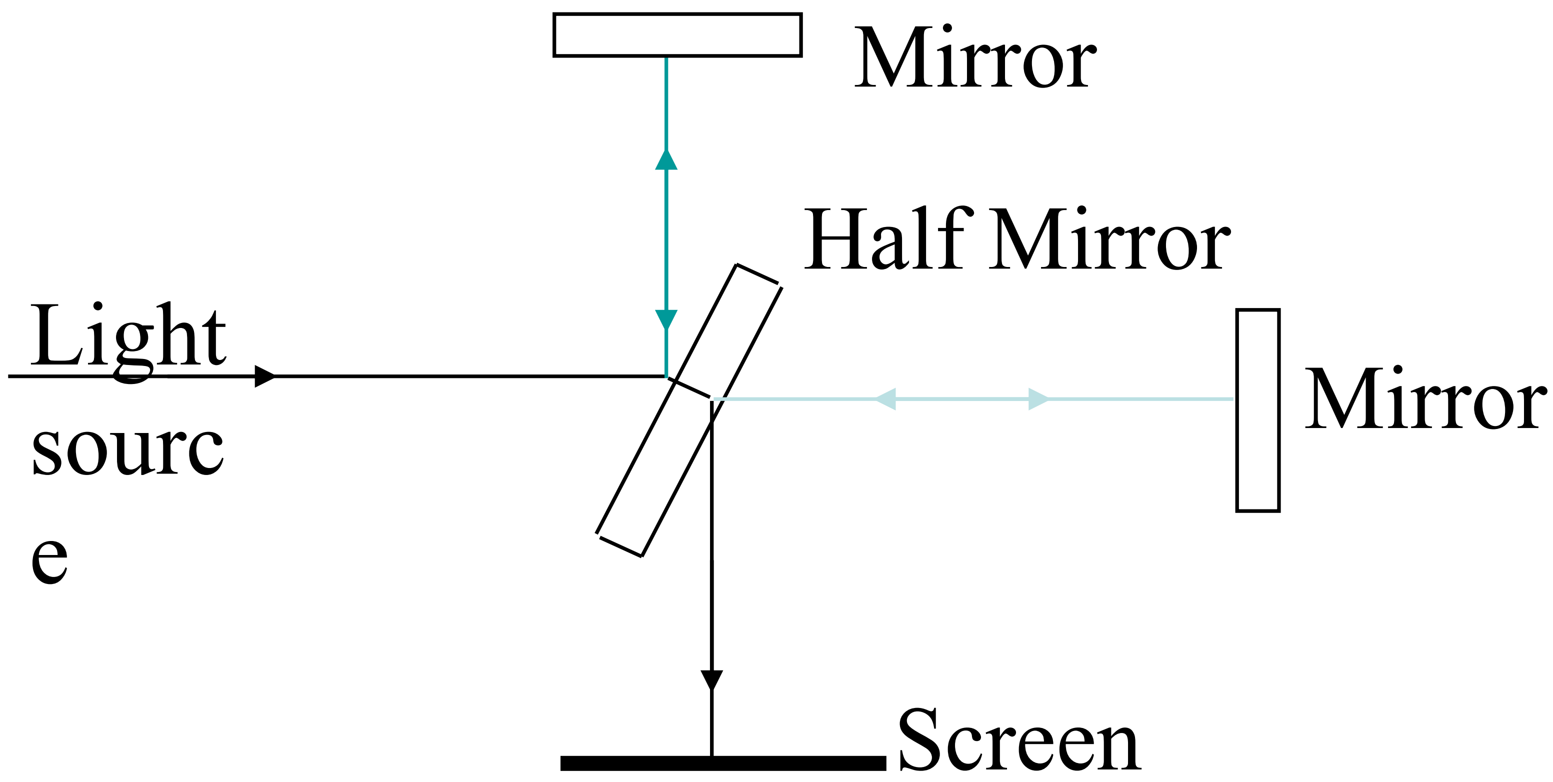
Michelson Interferometer

Split a beam with a Half Mirror, the use mirrors to recombine the two beams.



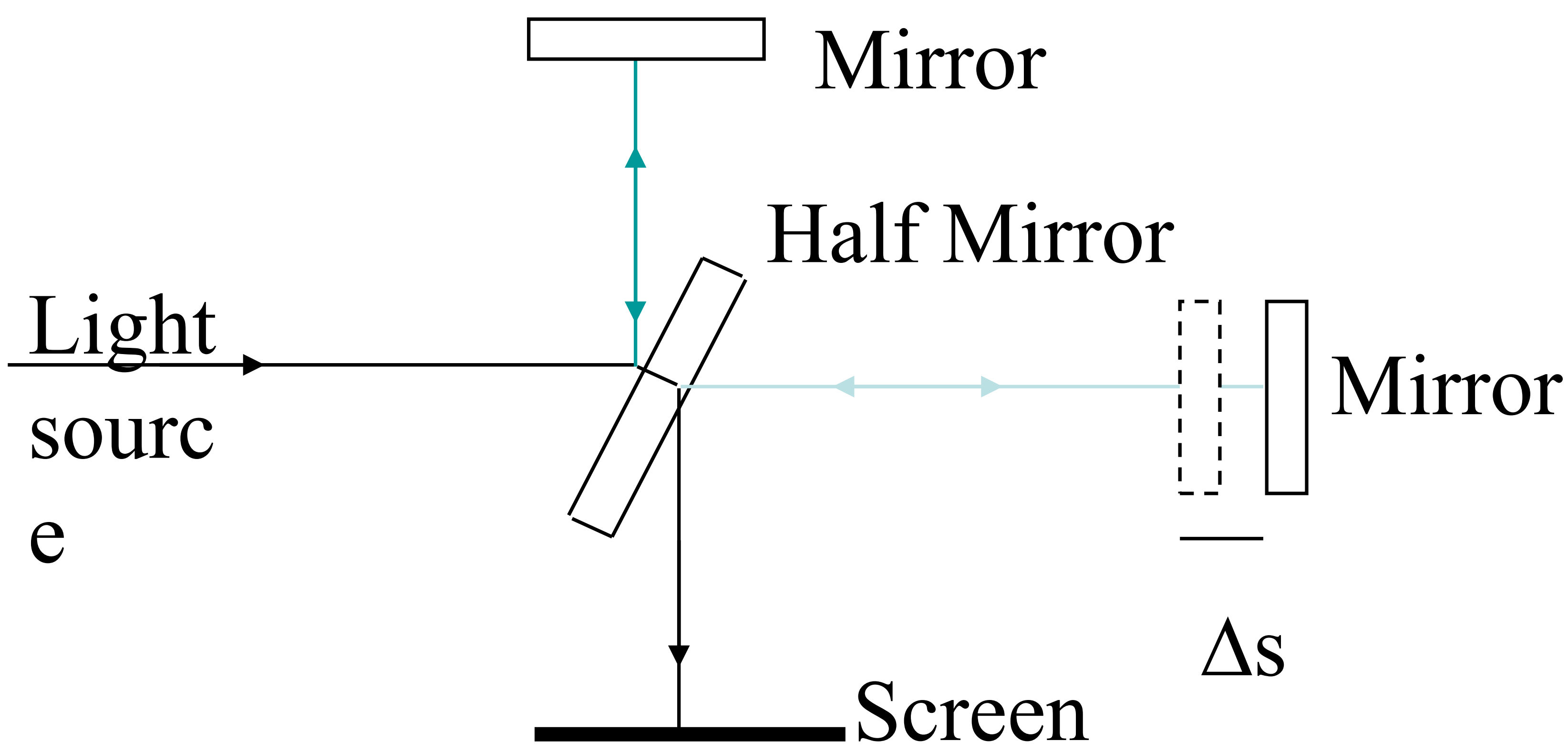
Michelson Interferometer

If the red beam goes the same length as the blue beam, then the two beams will constructively interfere and a bright spot will appear on screen.



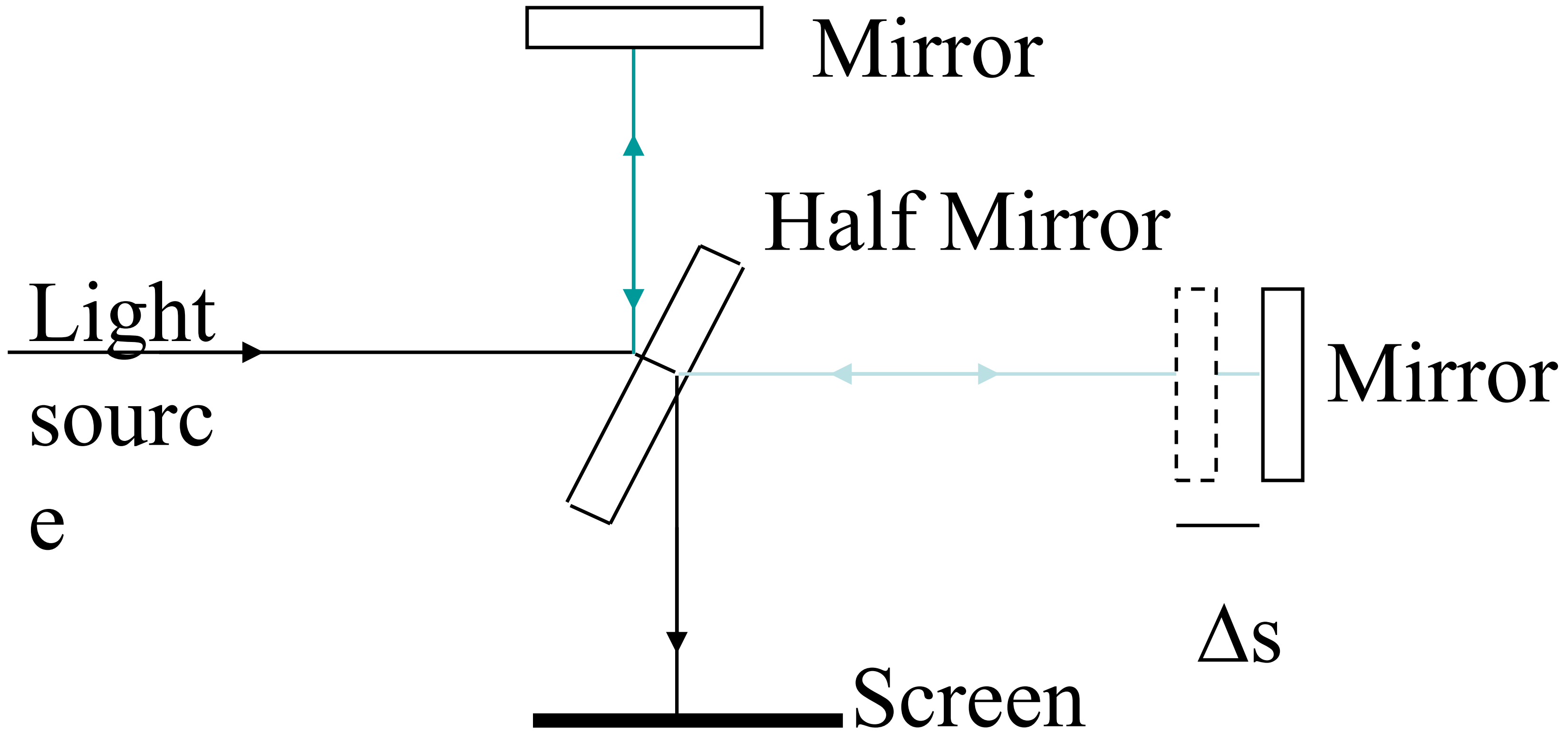
Michelson Interferometer

If the blue beam goes a little extra distance, Δs, the the screen will show a different interference pattern.



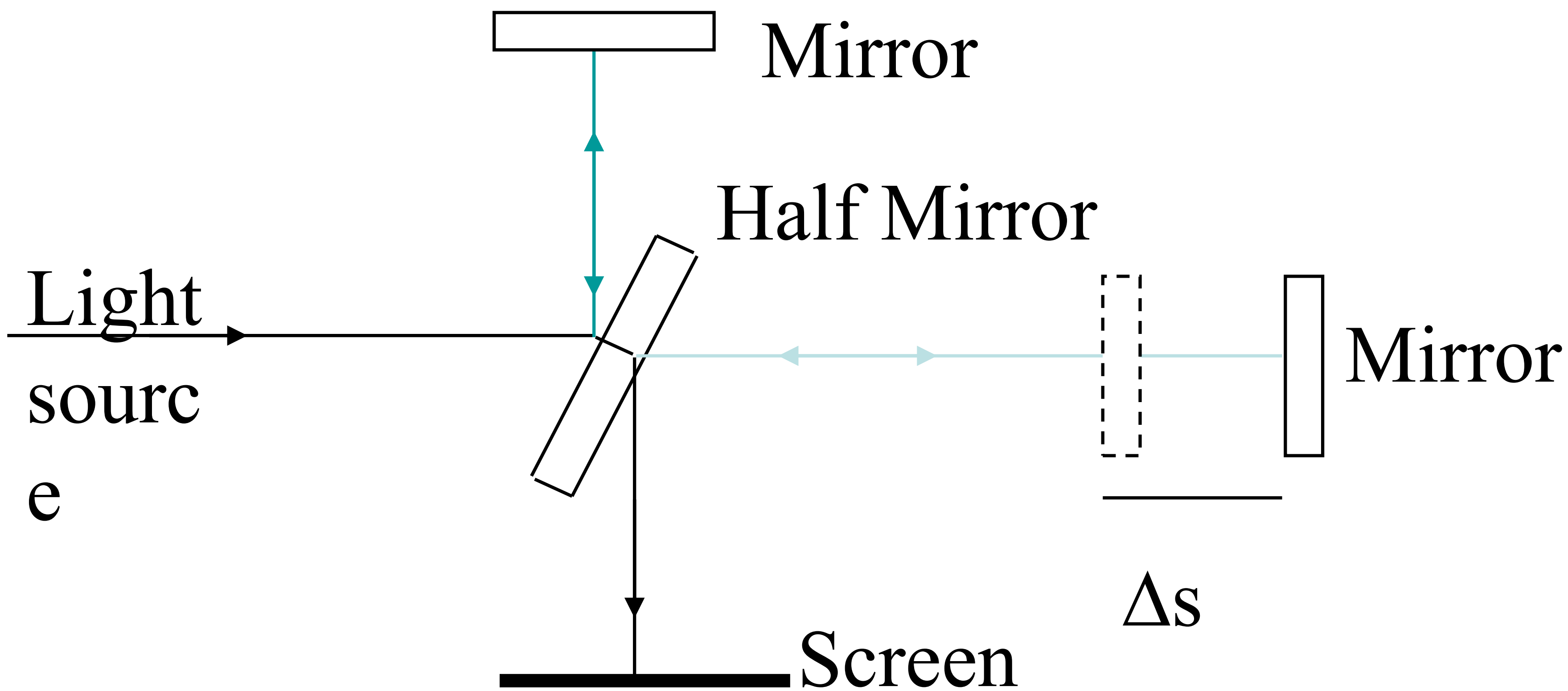
Michelson Interferometer

If $\Delta s = \lambda/4$, then the interference pattern changes from bright to dark.



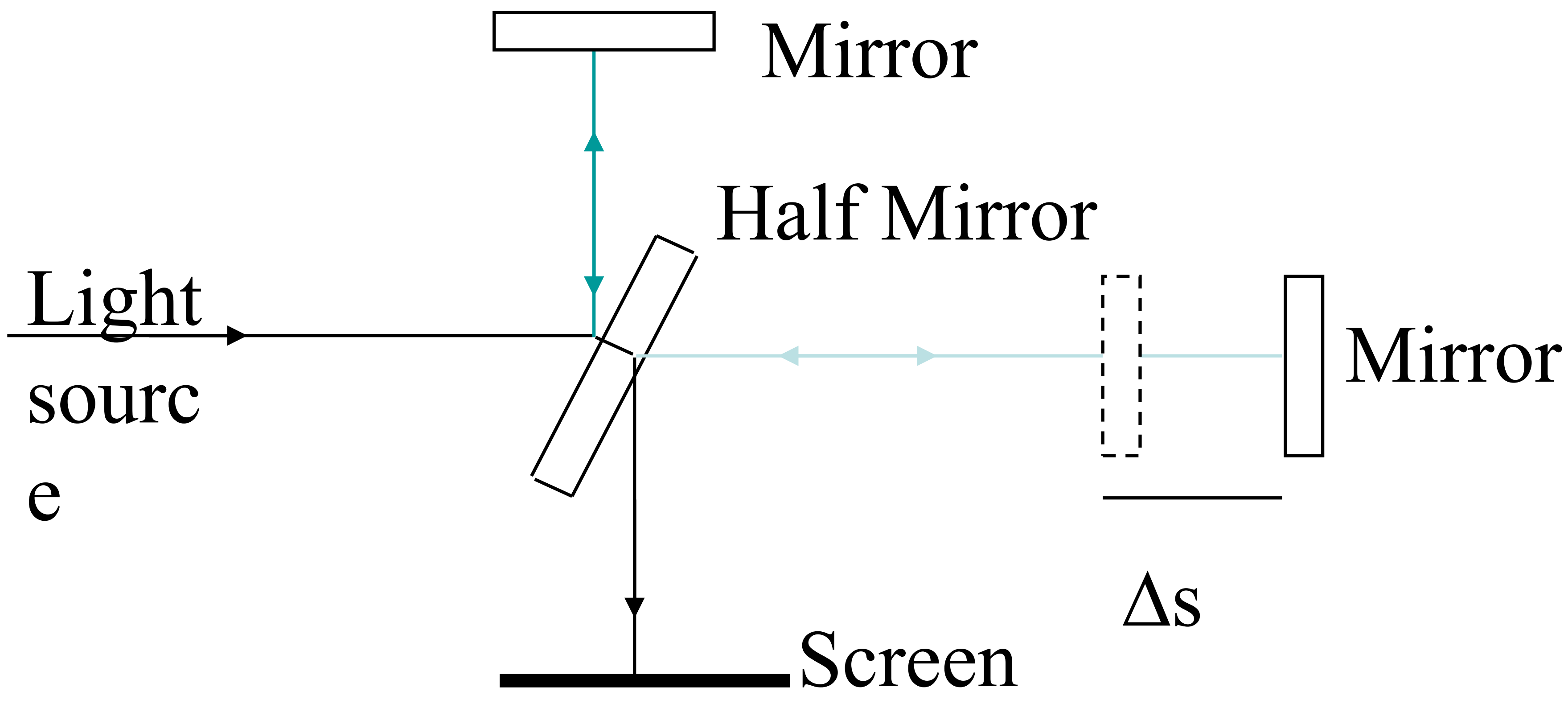
Michelson Interferometer

If $\Delta s = \lambda/2$, then the interference pattern changes from bright to dark back to bright (a fringe shift).



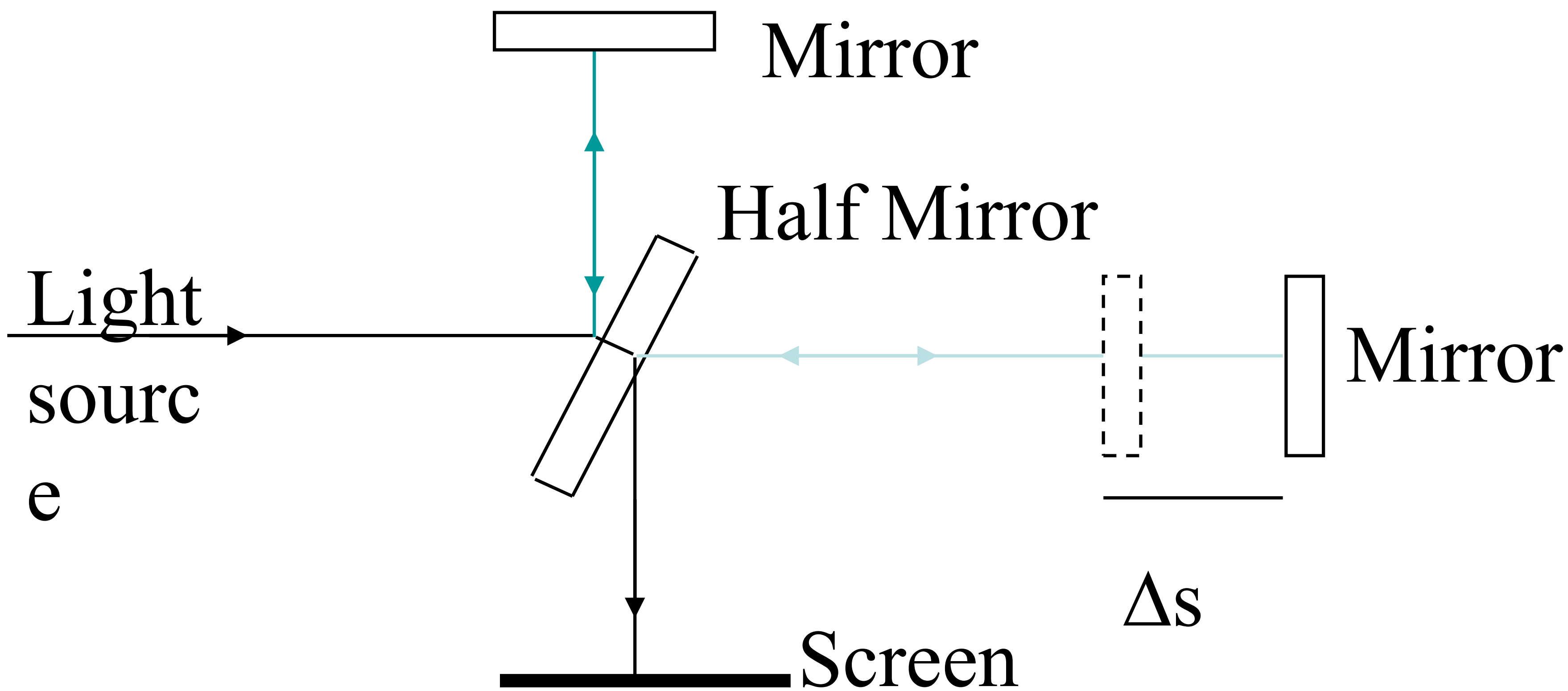
Michelson Interferometer

By counting the number of fringe shifts, we can determine how far Δs is!

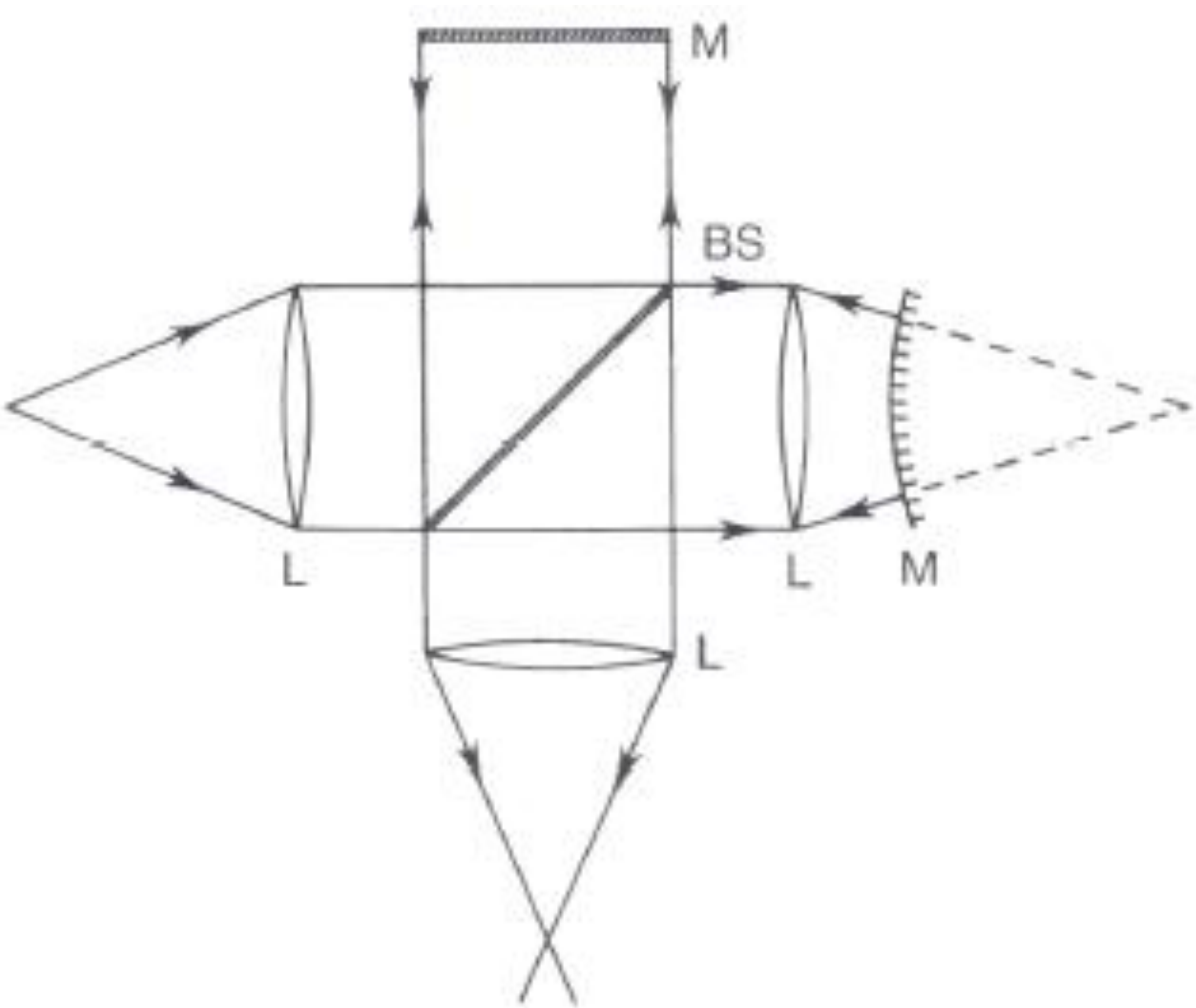


Michelson Interferometer

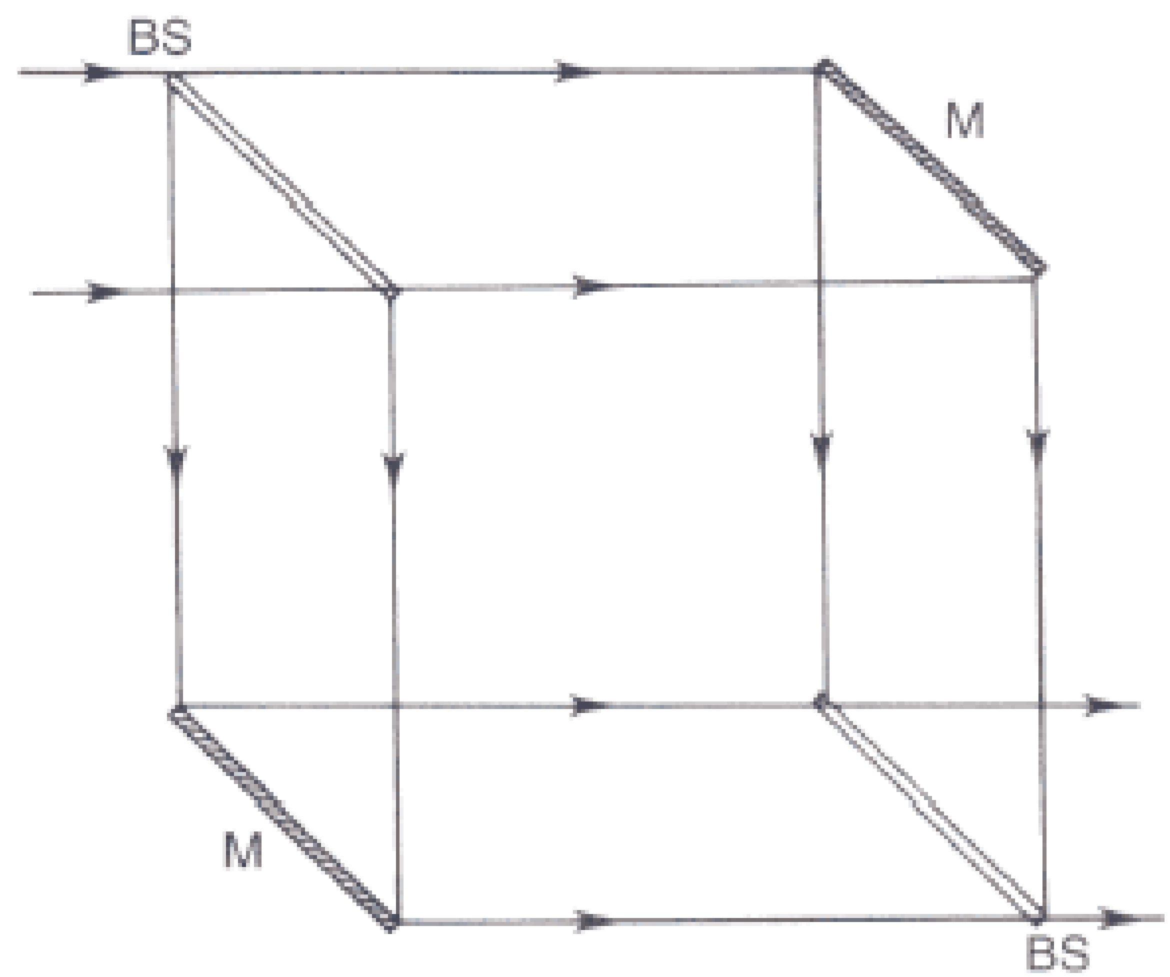
If we use the red laser (λ=632 nm), then each fringe shift corresponds to a distance the mirror moves of 316 nm (about 1/3 of a micron)!



Amplitude Division



(a)



(b)

- Twyman Green Interferometer
- Mach Zehnder Interferometer

Dual Frequency Interferometer

- We stated that two waves of different frequencies do not produce observable interference.
- By combining two plane waves

$$\psi_1 = e^{i2\pi[(z/\lambda_1) - \nu_1 t]}$$

$$\psi_2 = e^{i2\pi[(z/\lambda_2) - \nu_2 t]}$$

- The resultant intensity becomes

$$I = 2\left[1 + \cos 2\pi\left(\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)z - (\nu_1 - \nu_2)t\right)\right]$$

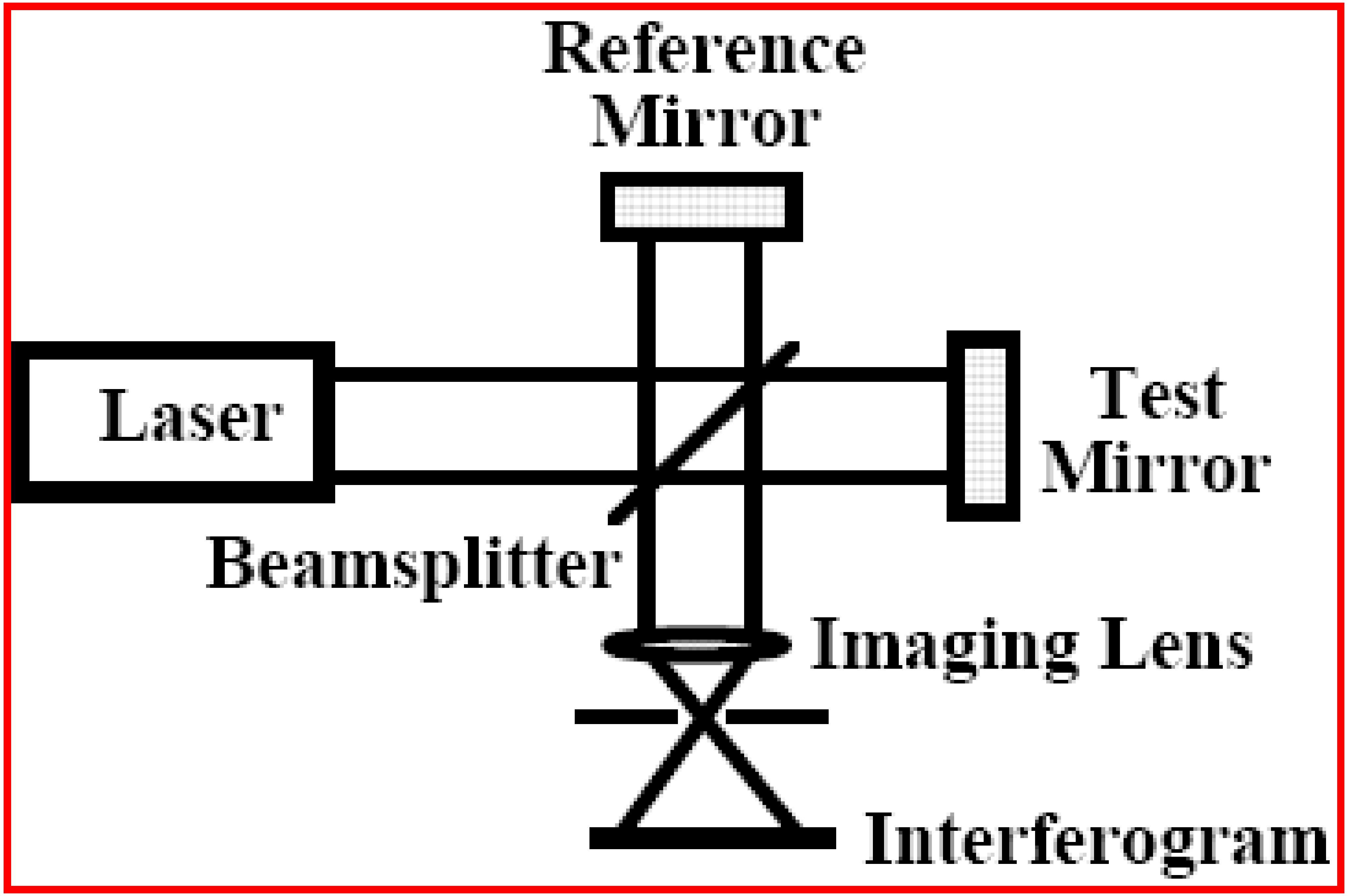
- If the frequency difference V_l - V_2 is very small and constant, this variation in I with time can be detected
- This is utilized in the dual-frequency Michelson interferometer for length measurement
- Also called as Heterodyne interferometer

Outline

- Interferometry Examples
- **Moire and Phase Shifting Interferometry**
- Theory
- Types of measurement
- Applications (form and stress measurement)
- **Theory of phase shifting**
- Types of phase shifting methods available,
- Errors associated with phase shifting

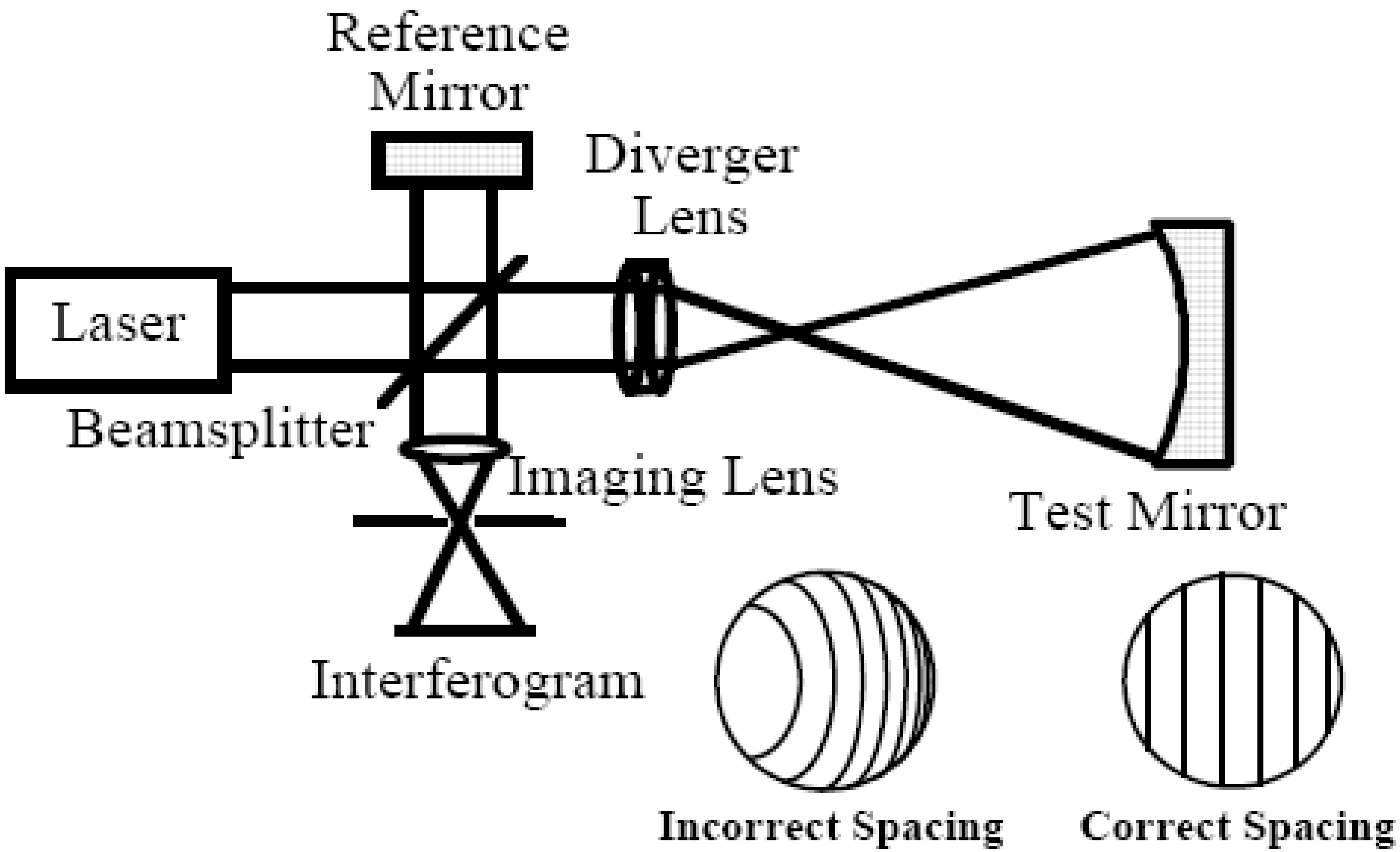
Twyman Green Interferometer

Flat Surfaces

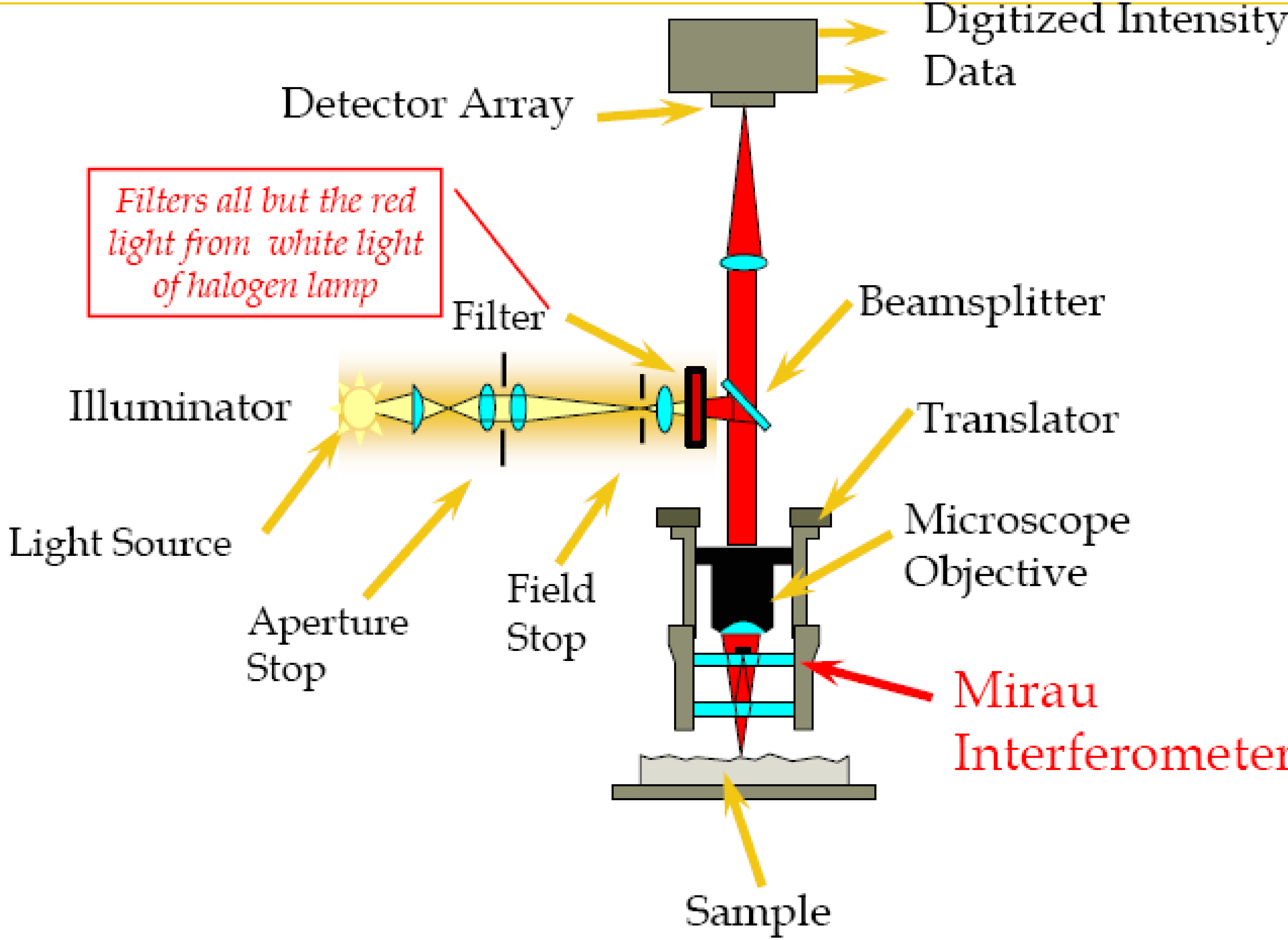


Twyman Green Interferometer

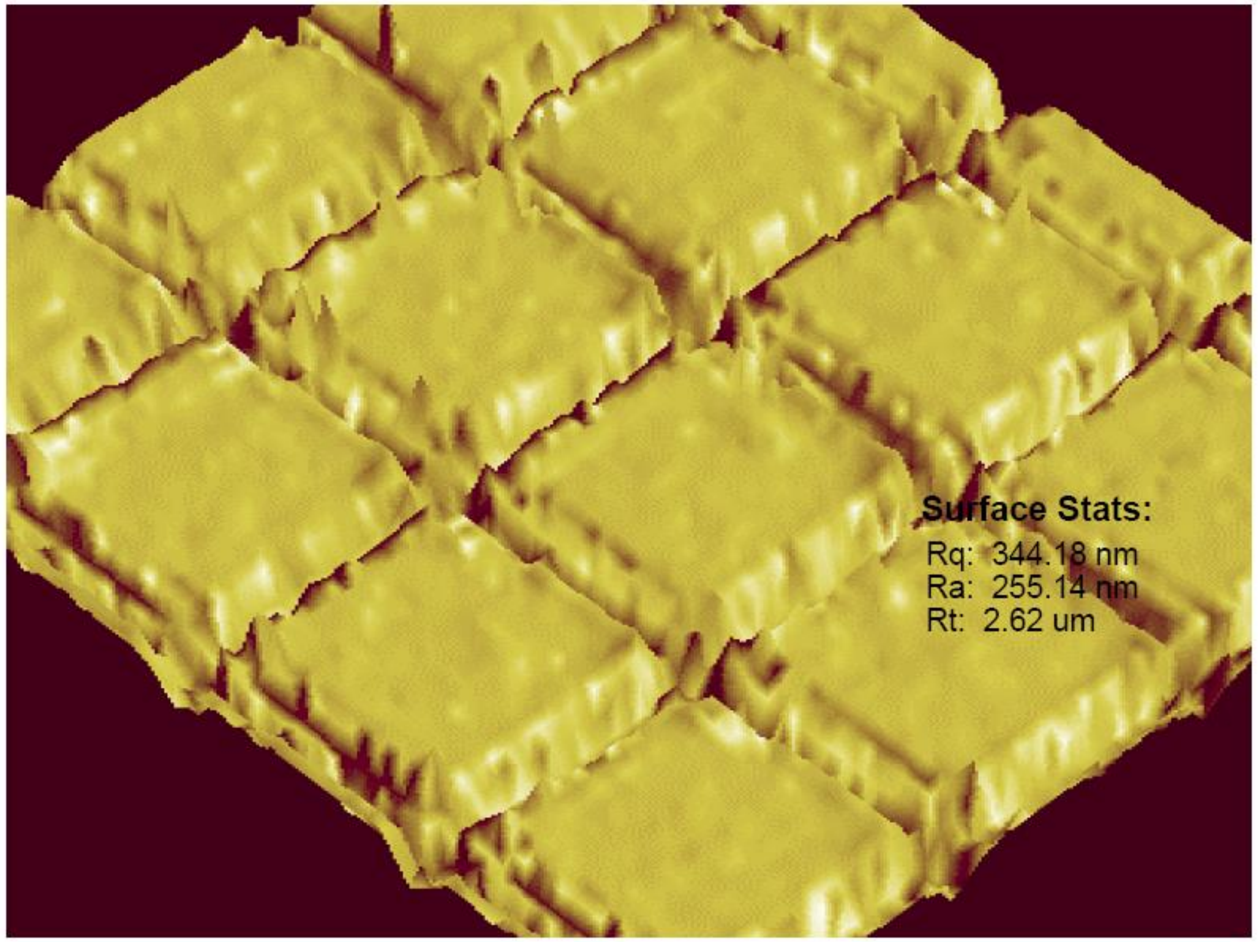
Spherical Surfaces



Mirau Interferometer

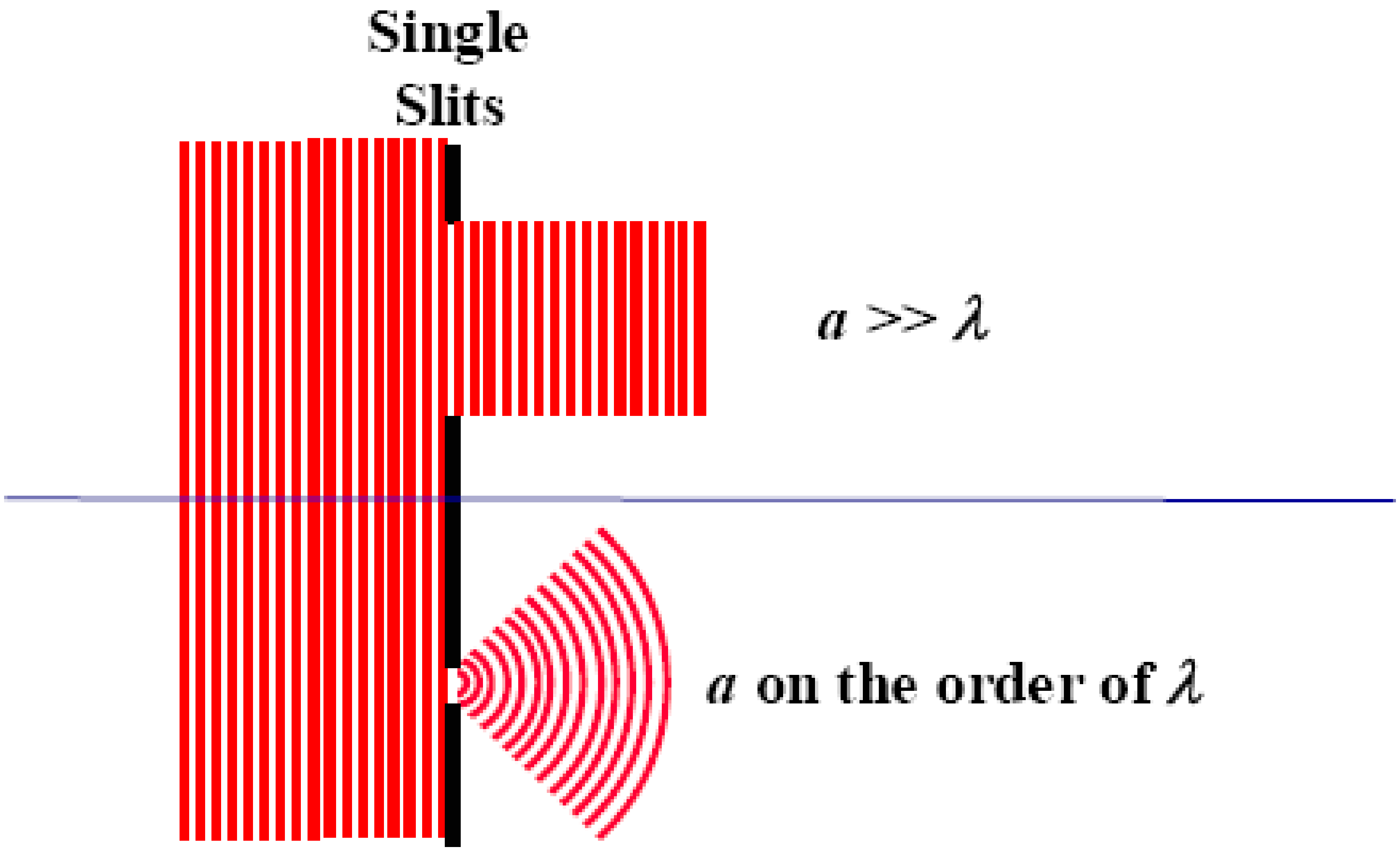


Mirau Interferometer



Diffraction

When slit size, a, is much greater than λ, the slit casts a shadow.

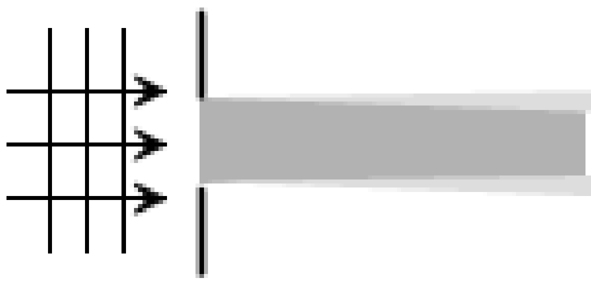


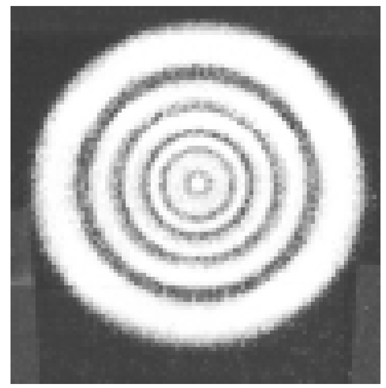
When slit size, a, is on the order of λ, the light passing through the slit diffracts.

Diffraction

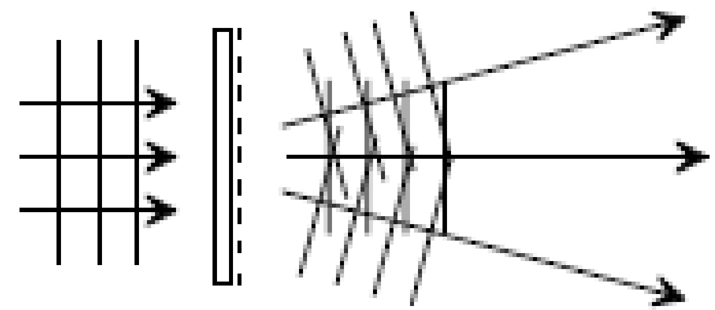
Types of diffraction

Fresnel diffraction

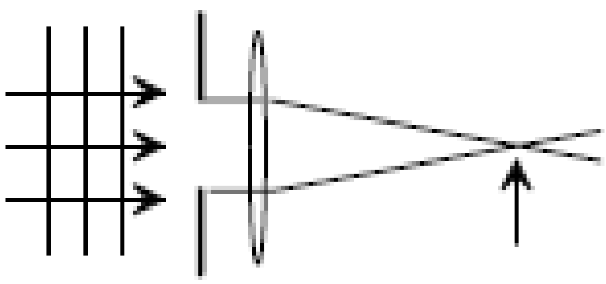


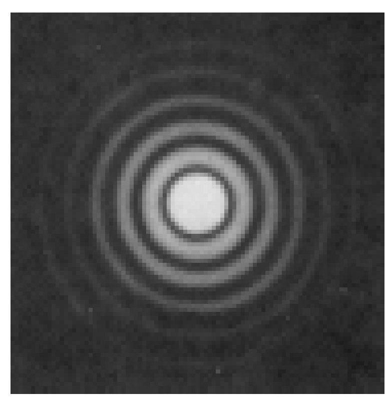


Grating: periodic structure – diffraction orders

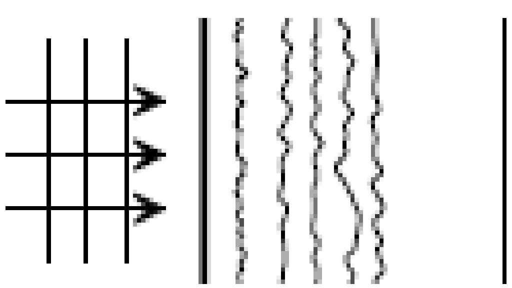


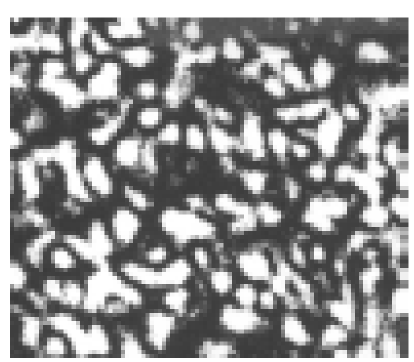
Fraunhofer diffraction - Airyn pattern





Diffraction from rough surface - speckle



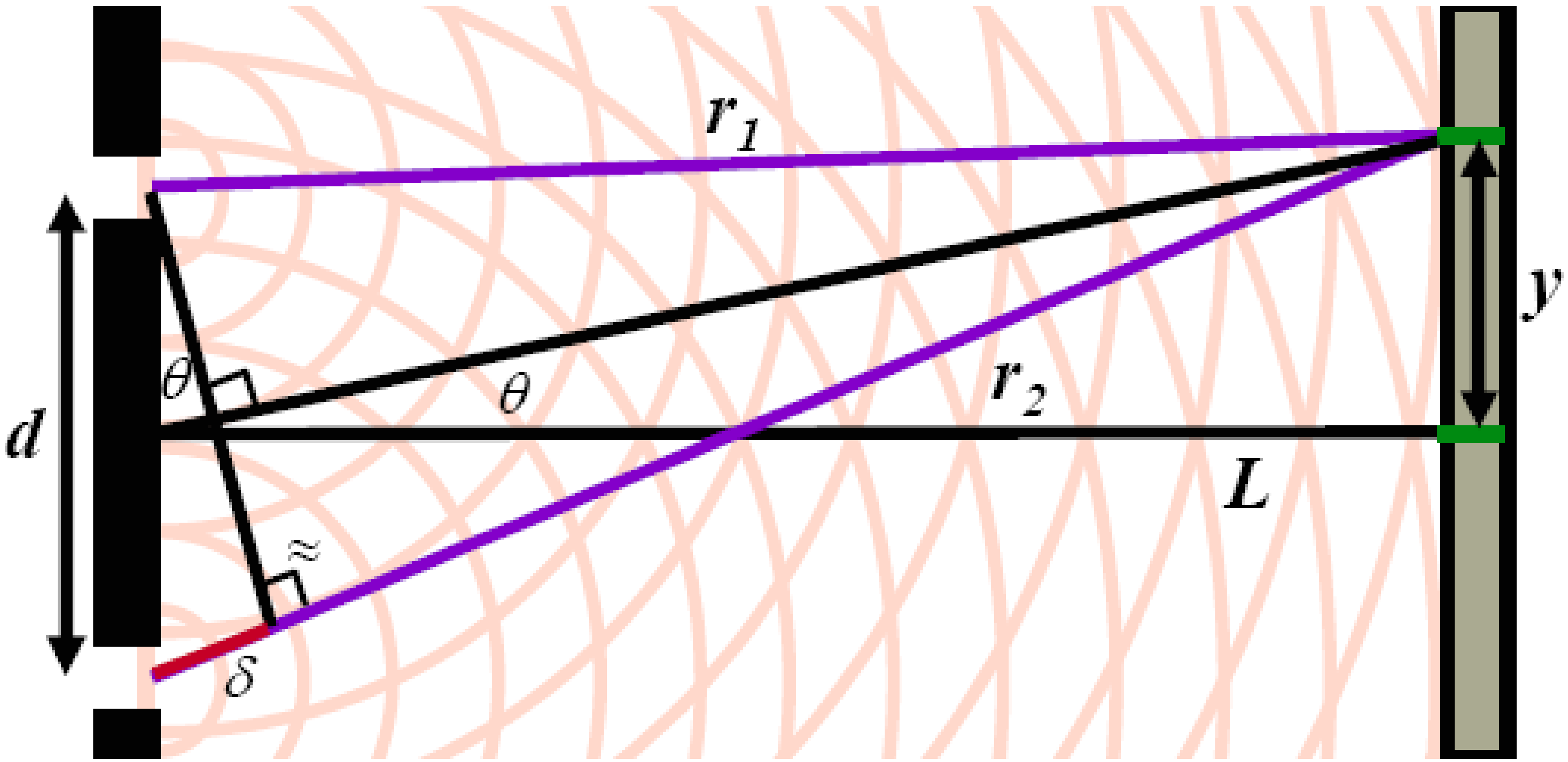


History:

Grimaldi, 1665	described the phenomenon
Huygens, 1678	wave theory of light
Fresnel, 1818	intuitive explanation
Kirchhoff, 1882	mathematical formulation

Diffraction

Double Slit



$$\boxed{\delta = r_2 - r_1 \approx d\sin(\theta)}$$

$L >> d$

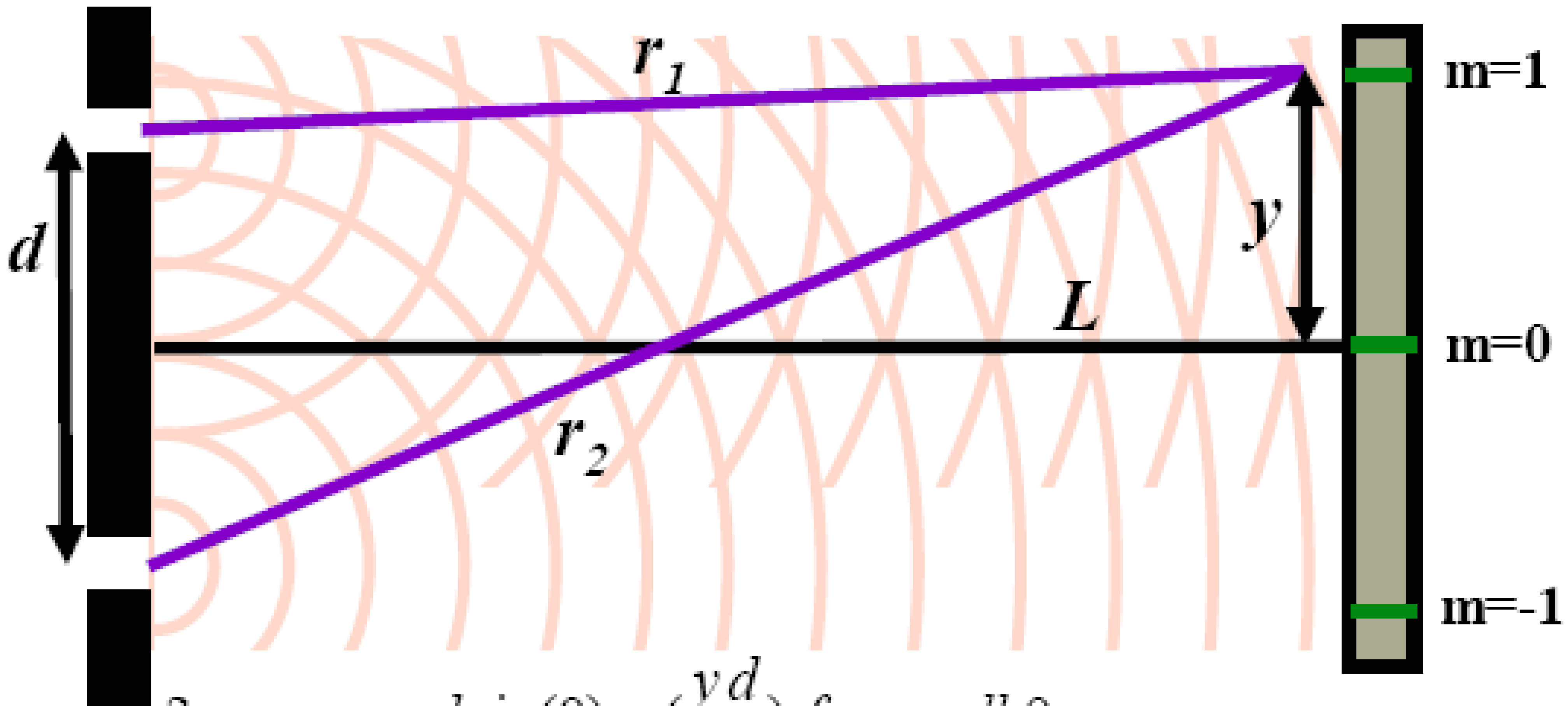
$L >> y$

$$\text{For } y << L: \quad \sin(\theta) \approx \theta \approx \frac{y}{L}$$

$$\text{So,} \quad \boxed{\delta = \frac{yd}{L}}$$

Diffraction

Double Slit



$$\delta = r_2 - r_1 \approx d\sin(\theta) \approx \left(\frac{y\,d}{L}\right) \; for\ small\ \theta$$

$L >> y$

$L >> d$

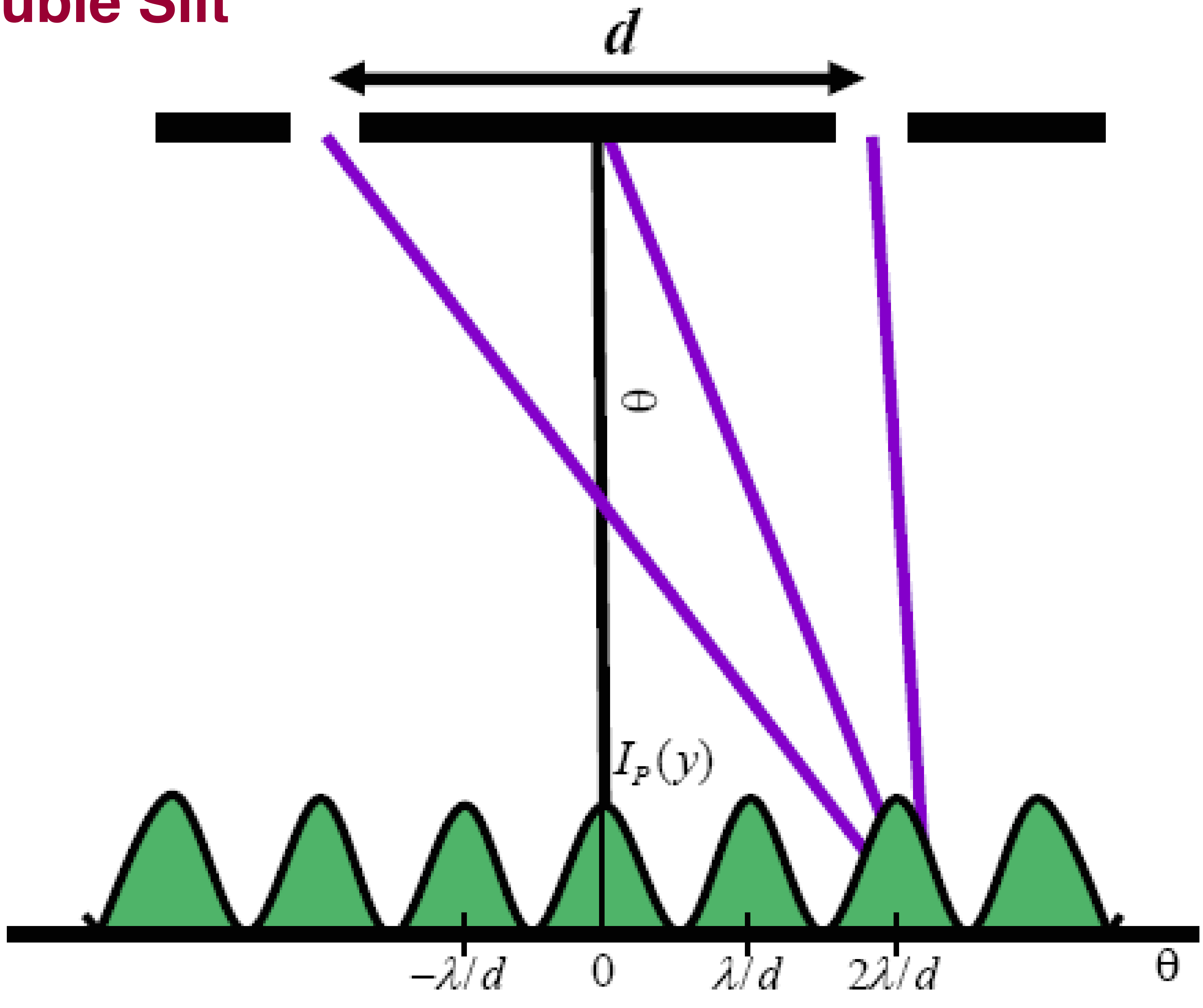
$d >> \lambda$

Bright Fringes : $\boxed{\delta = m\lambda}$ $\boxed{y_m = \frac{\lambda L}{d} m}$

Dark Fringes : $\boxed{\delta = (m + 1/2)\lambda}$ $\boxed{y_m = \frac{\lambda L}{d}(m + 1/2)}$

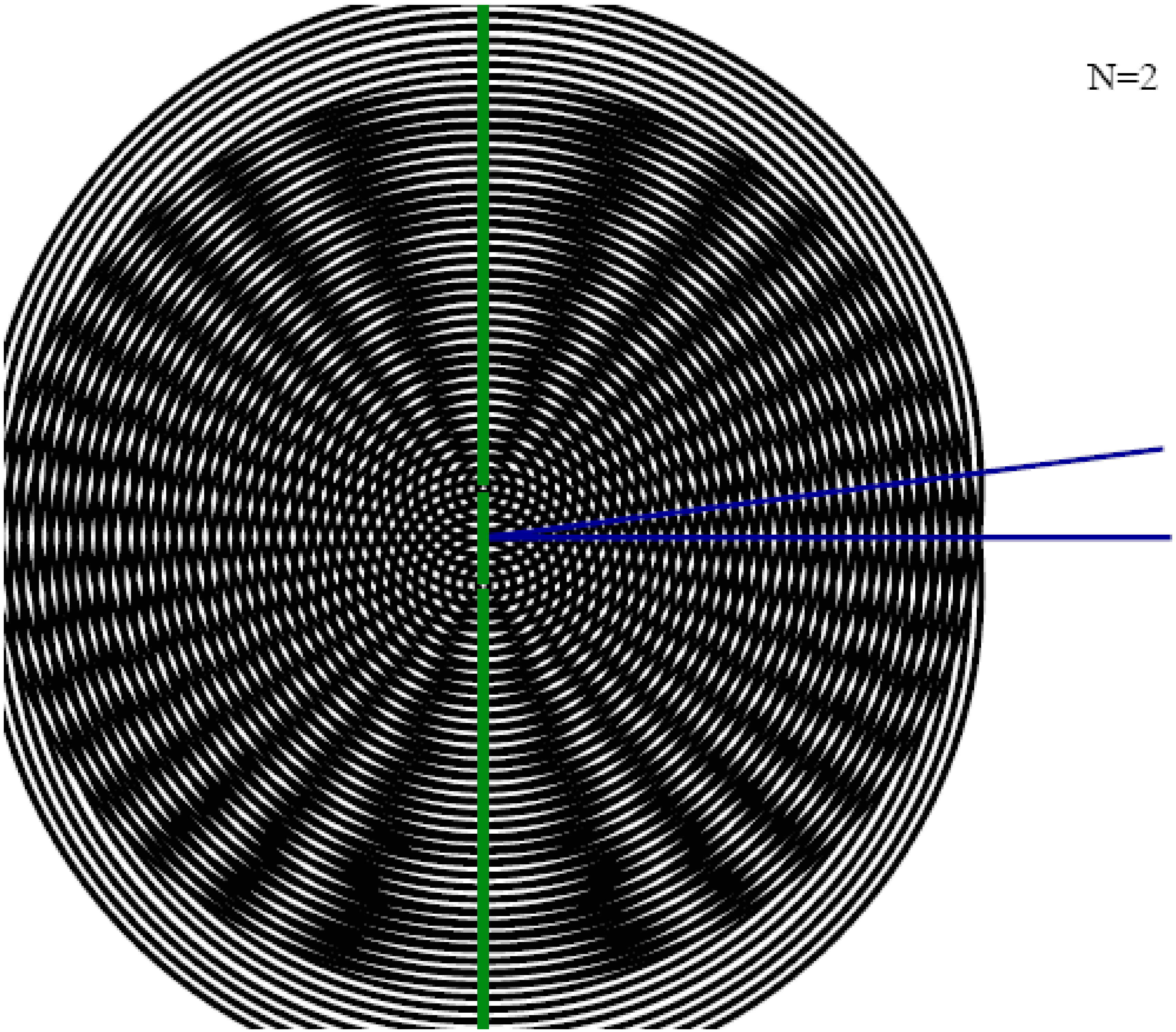
Diffraction

Double Slit



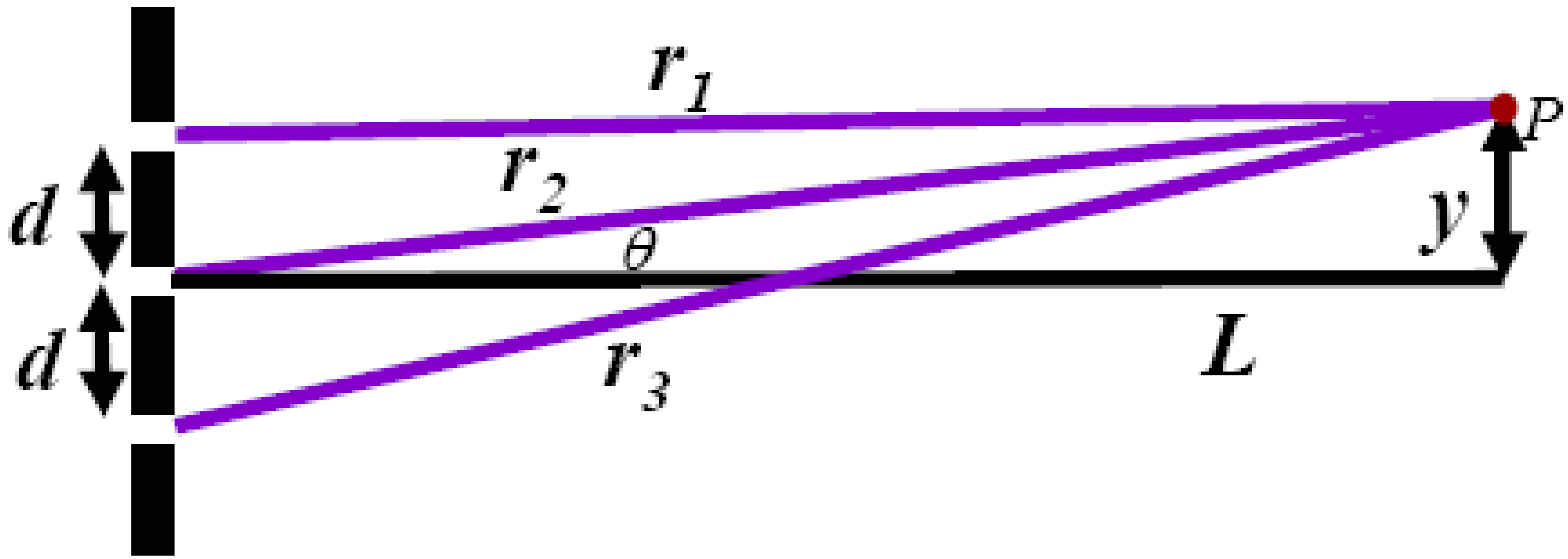
Diffraction

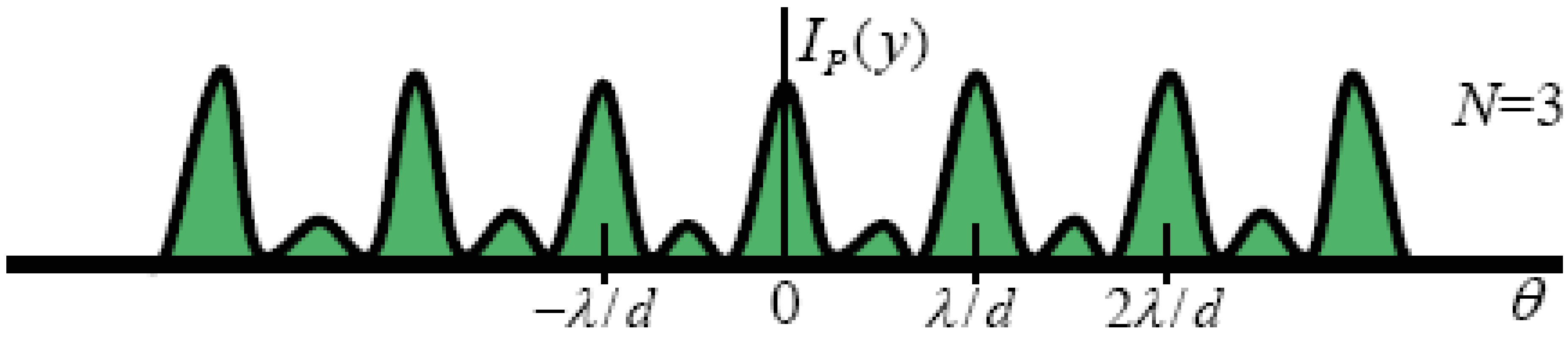
Double Slit



Diffraction

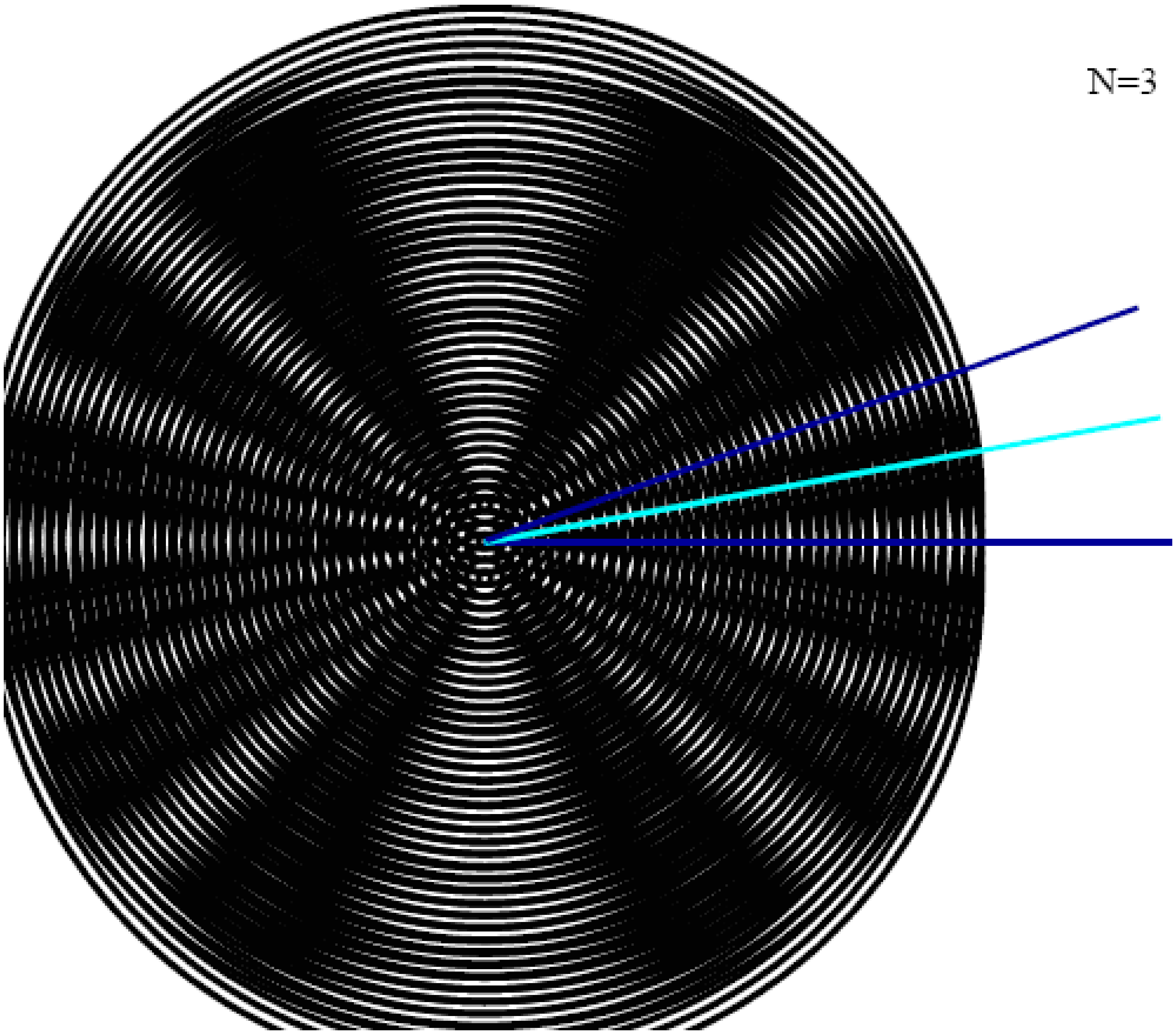
Triple Slit





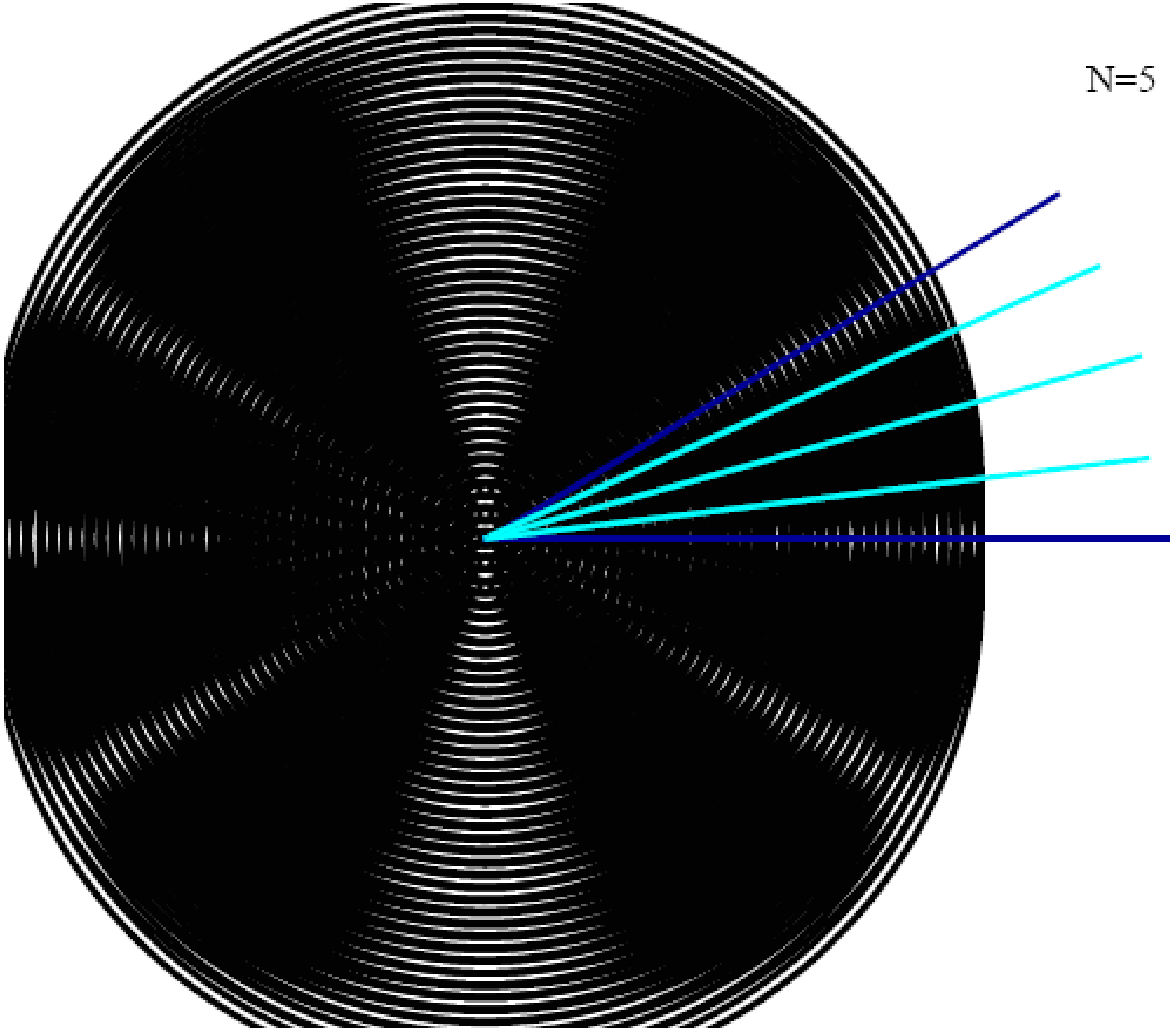
Diffraction

Triple Slit



Diffraction

Multiple Slit

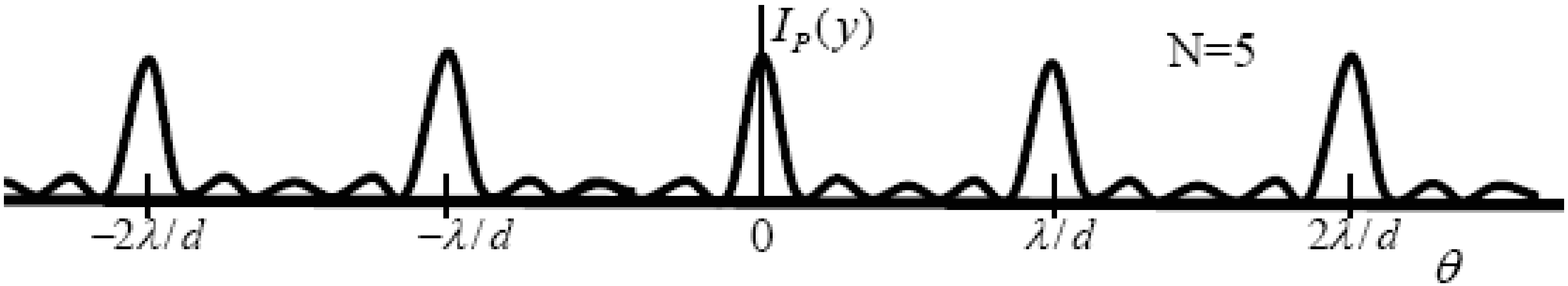


Diffraction

Multiple Slit

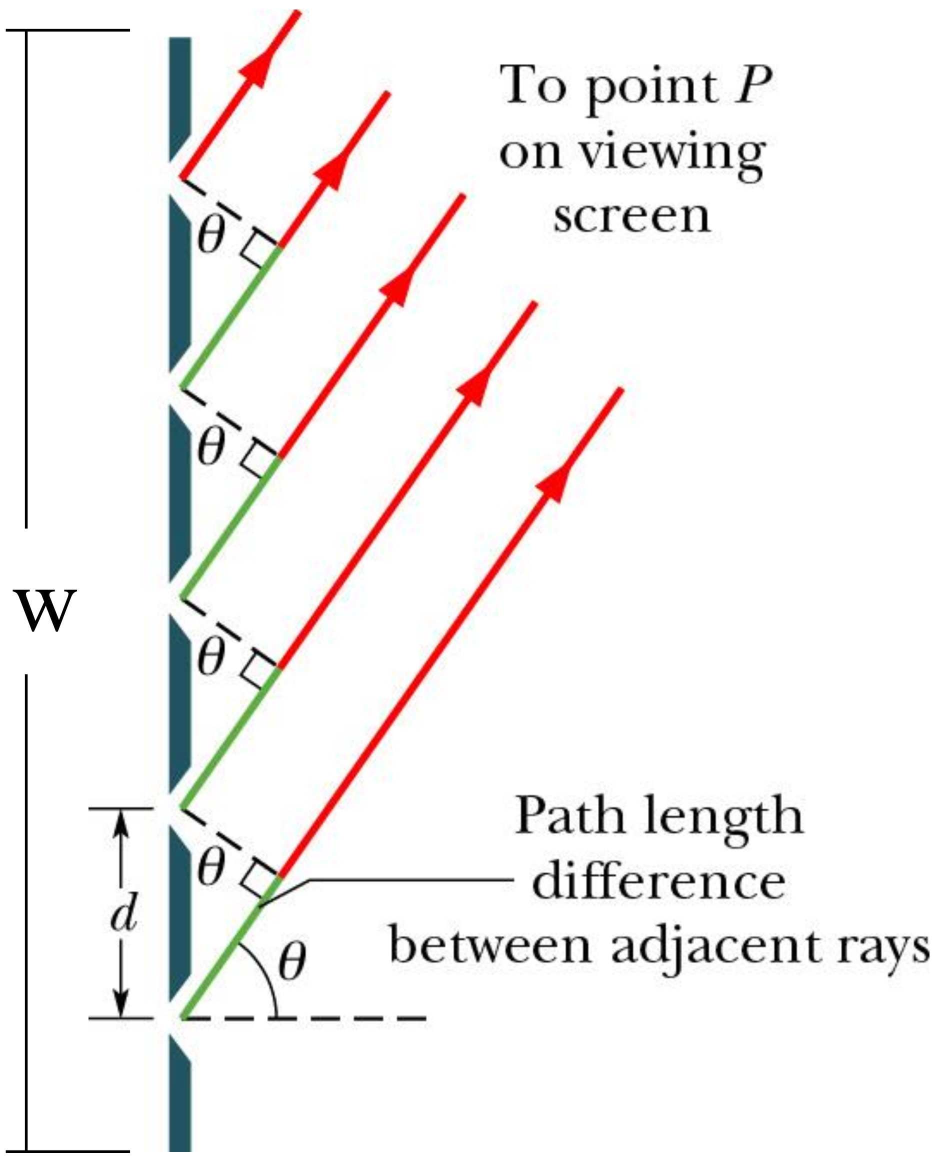
From N slits, with a spacing of d between adjacent slits:

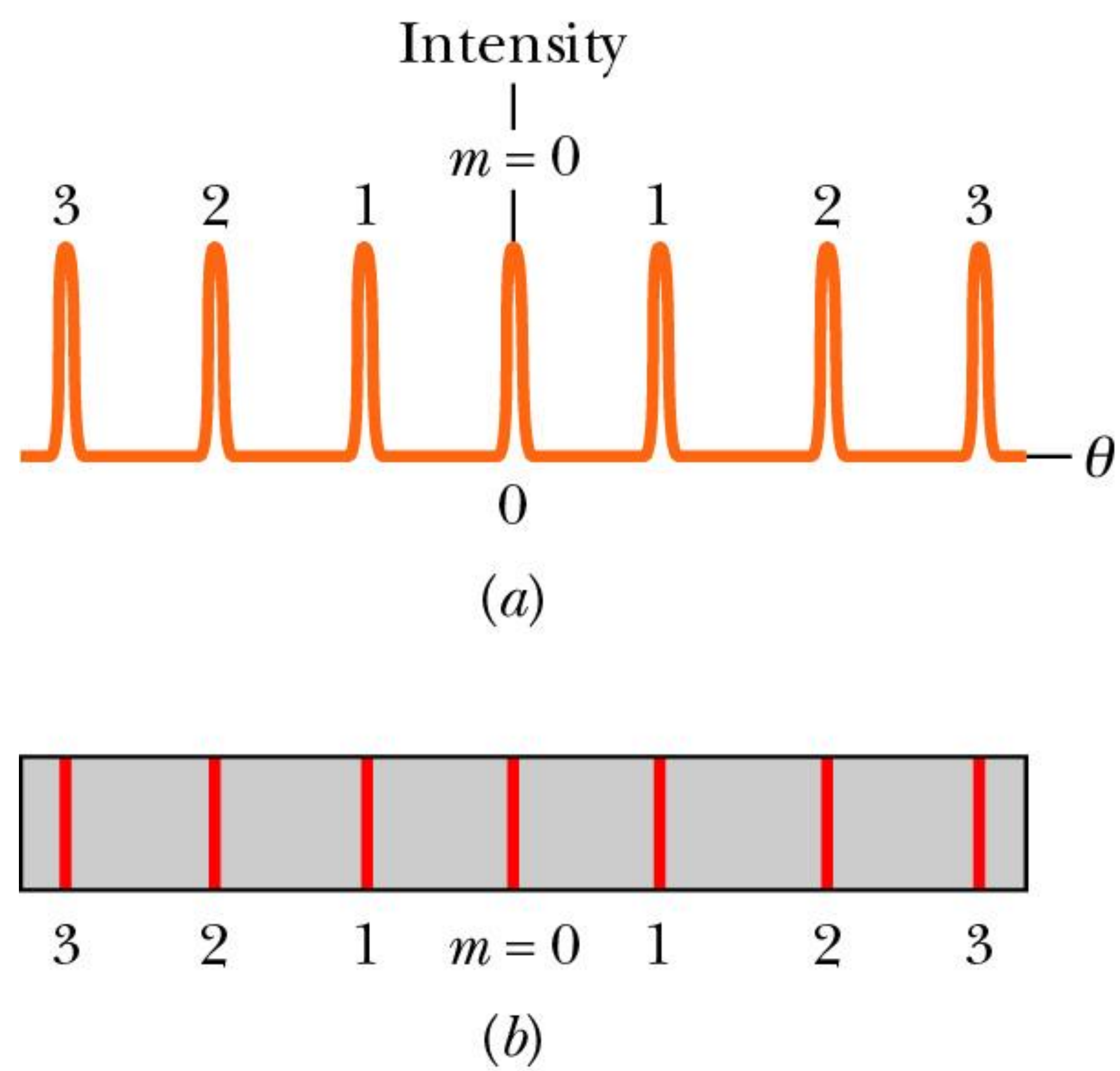
- *Primary maxima at $\theta = m\lambda/d$, where m (the order) is an integer.*
- *N-2 secondary maxima between adjacent primary maxima*
- *The width of the maxima is approximately $\lambda/(Nd)$.*
- *The ratio of the primary maxima to secondary maxima increases with increasing N.*



Diffraction

Grating -- N slits or rulings



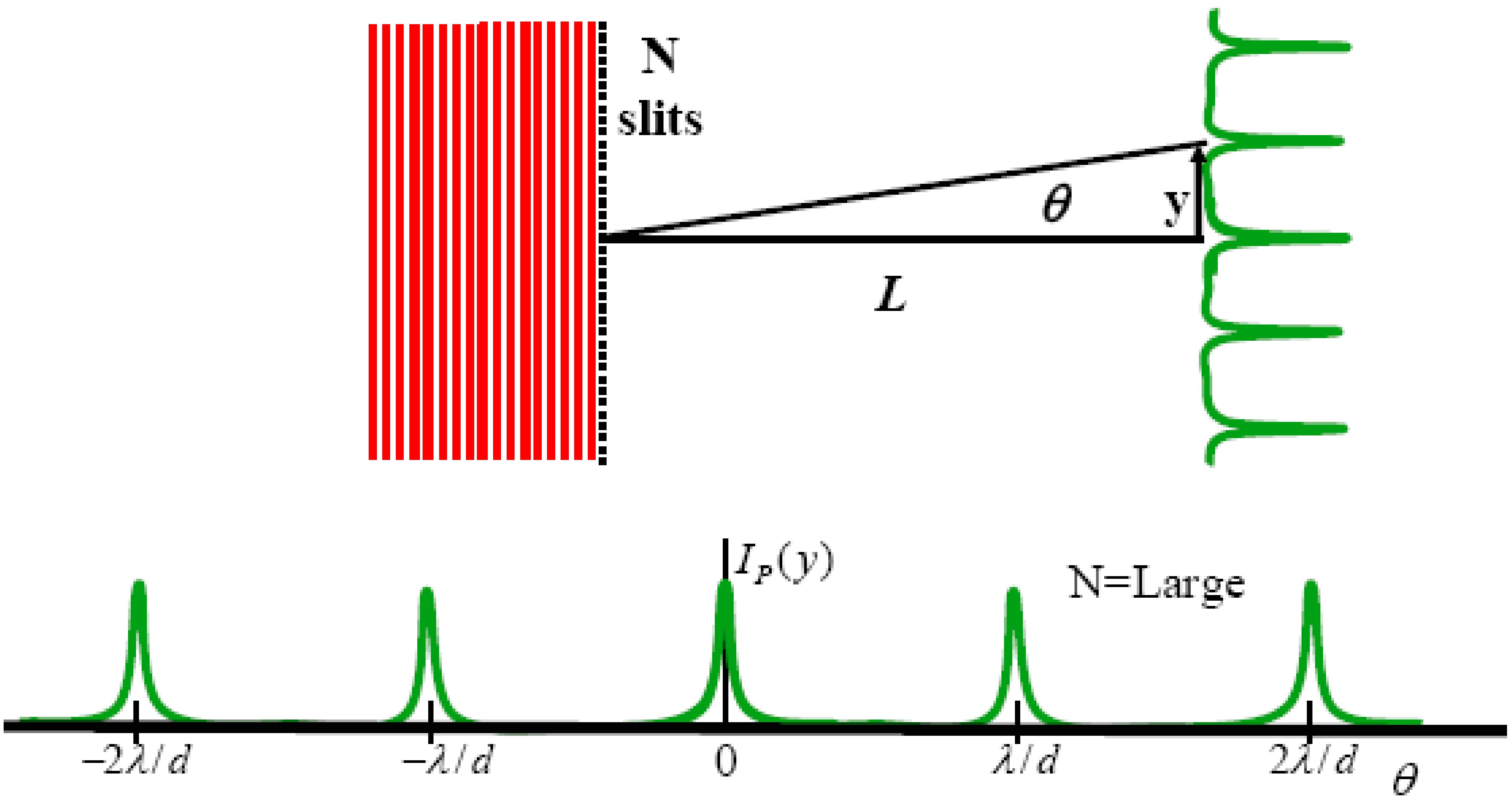


$$d \sin\theta = m\lambda \quad \mathrm{m} = 0,1,2,3..$$

$d = w/N$ where w is the entire width of the grating

Diffraction

Grating -- N slits or rulings



The width of each maxima is approximately $\lambda/(Nd)$

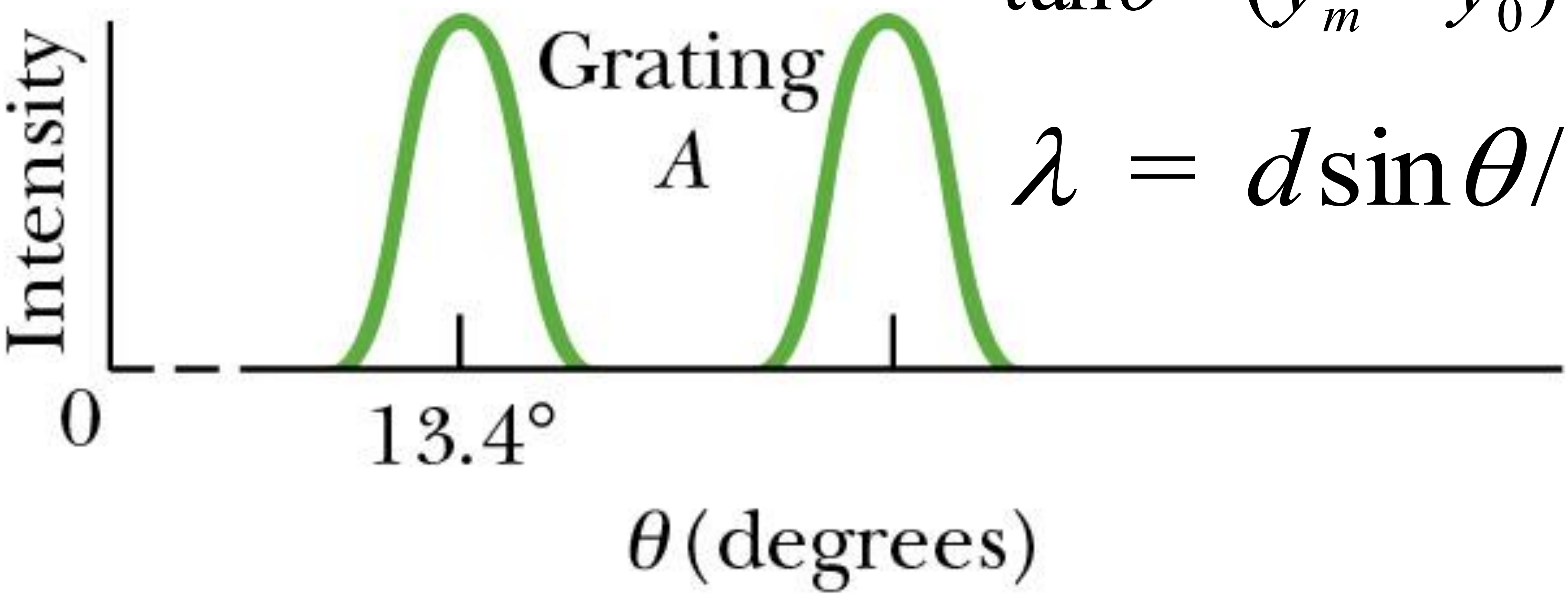
Diffraction Gratings

Measure Wavelength of Light

Measure angles of diffracted lines with a spectroscope using formula below. Then relate to wavelength

$$\tan\theta = (y_m - y_0)/D$$

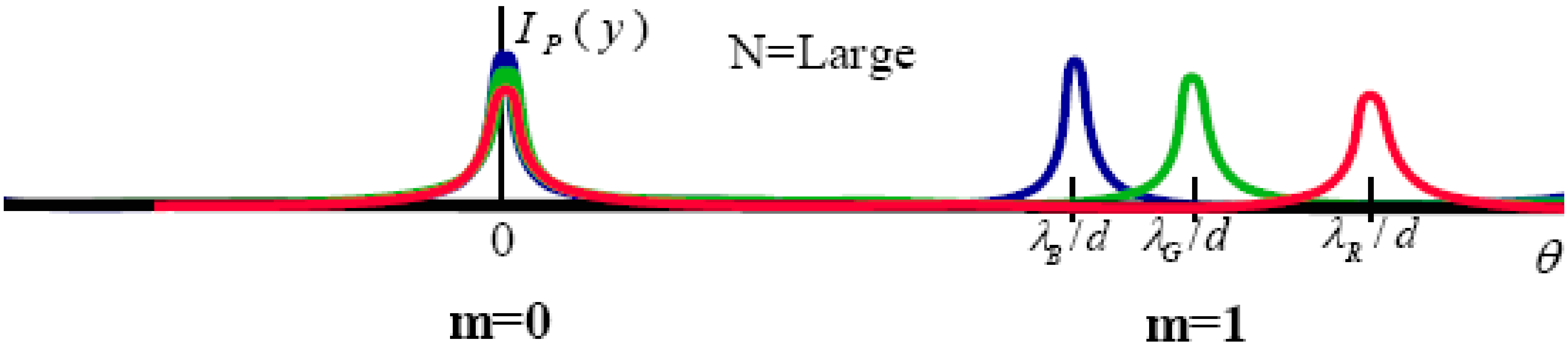
$$\lambda = d\sin\theta/m$$



Diffraction Gratings

Measure Wavelength of Light

If very different wavelengths illuminate a diffraction grating



For each wavelength

$$\boxed{\theta_\lambda = \frac{\lambda}{d}}$$

Higher wavelengths diffract more

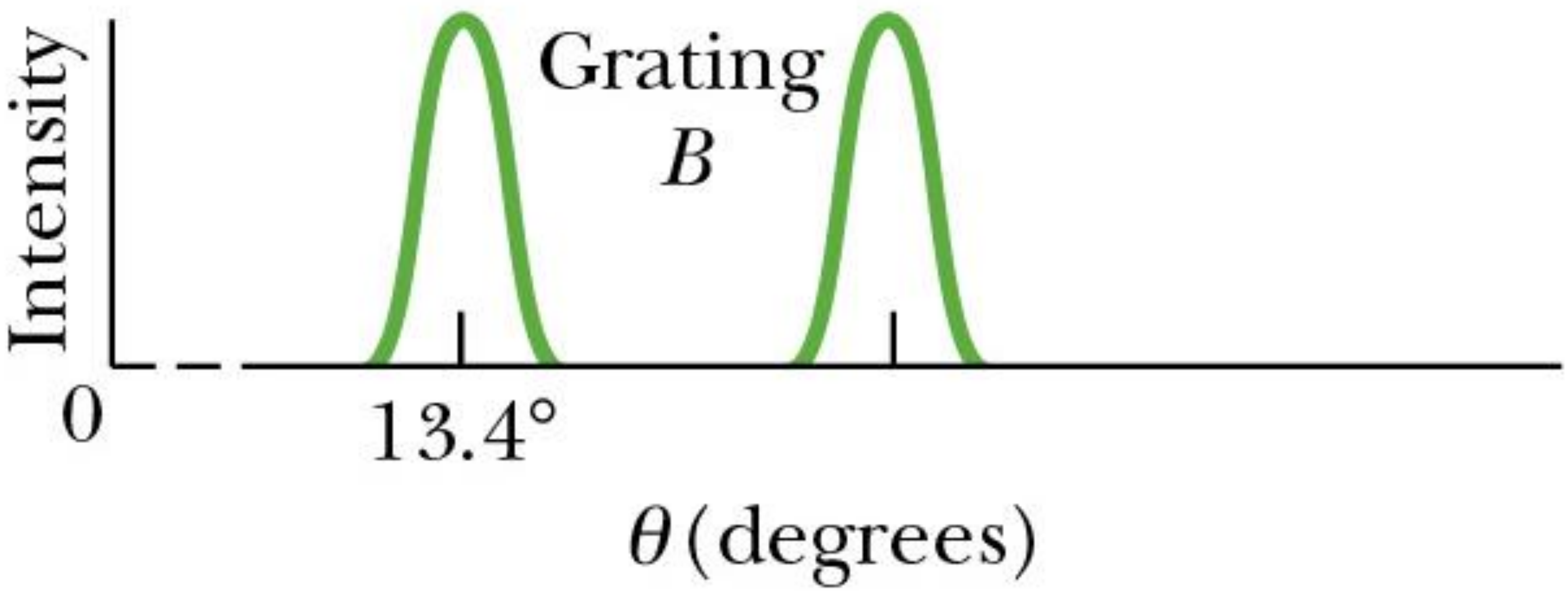
Diffraction Gratings

Resolving Power

Resolving power of grating.

Measure of the narrowness of lines

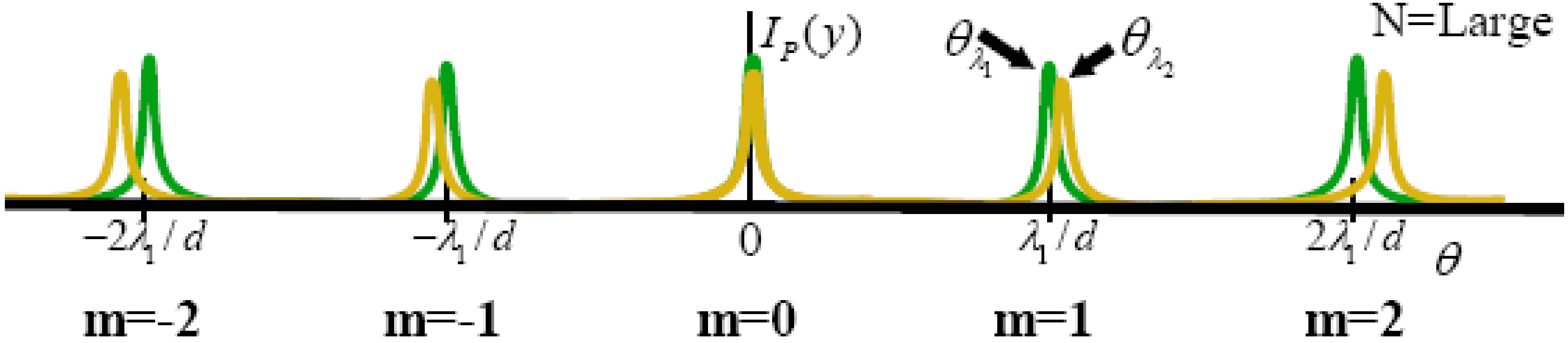
$$R = \lambda / \Delta\lambda = Nm$$



Diffraction Gratings

Resolving Power

If two nearly equal wavelengths illuminate a diffraction grating



The two wavelengths are resolved when

$$\boxed{\Delta\theta_{res} = \theta_{\lambda_2} - \theta_{\lambda_1}}$$

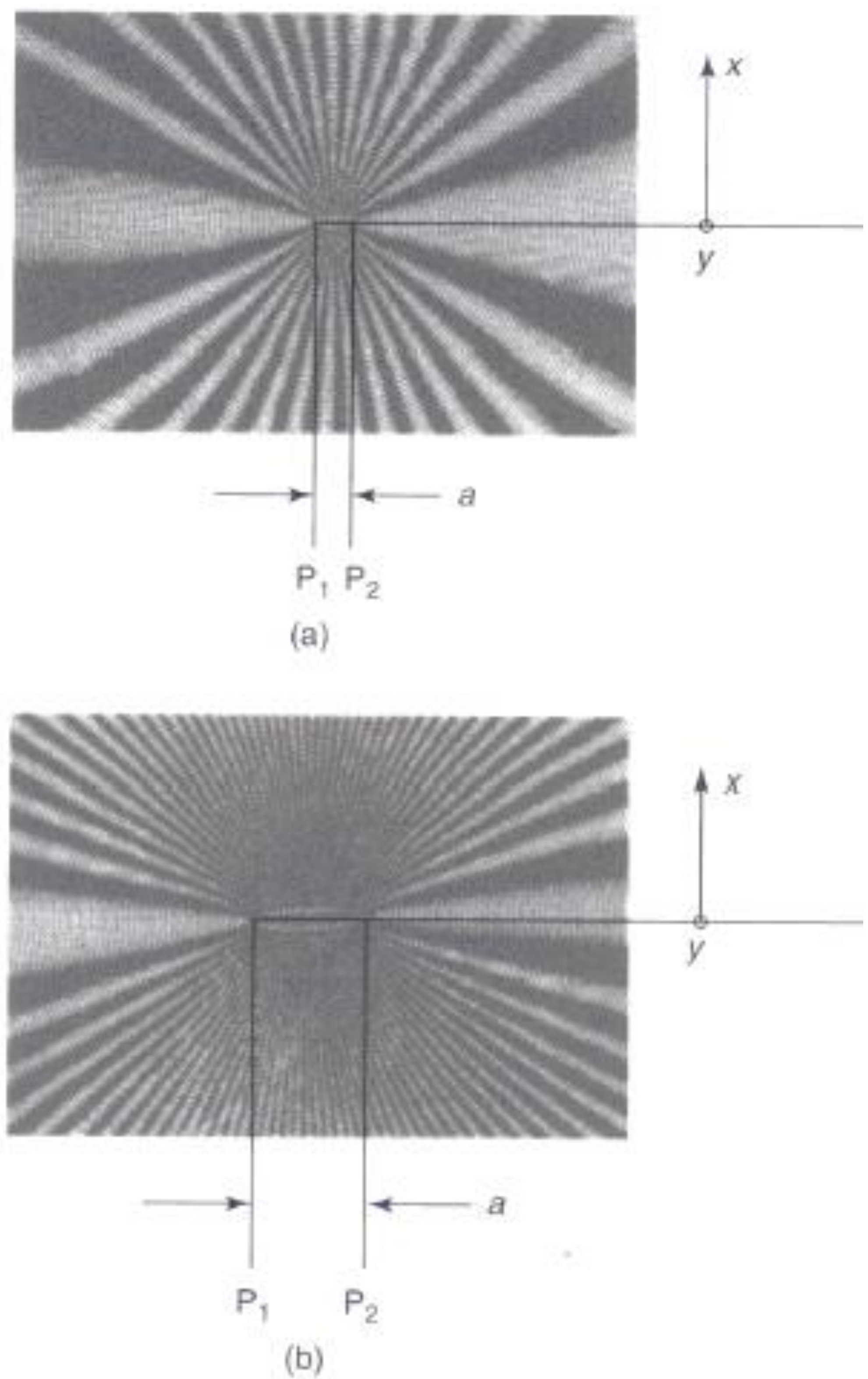
$$\boxed{\left(\frac{\lambda_1}{Nd}\right) = m\left(\frac{\lambda_2}{d} - \frac{\lambda_1}{d}\right)}$$

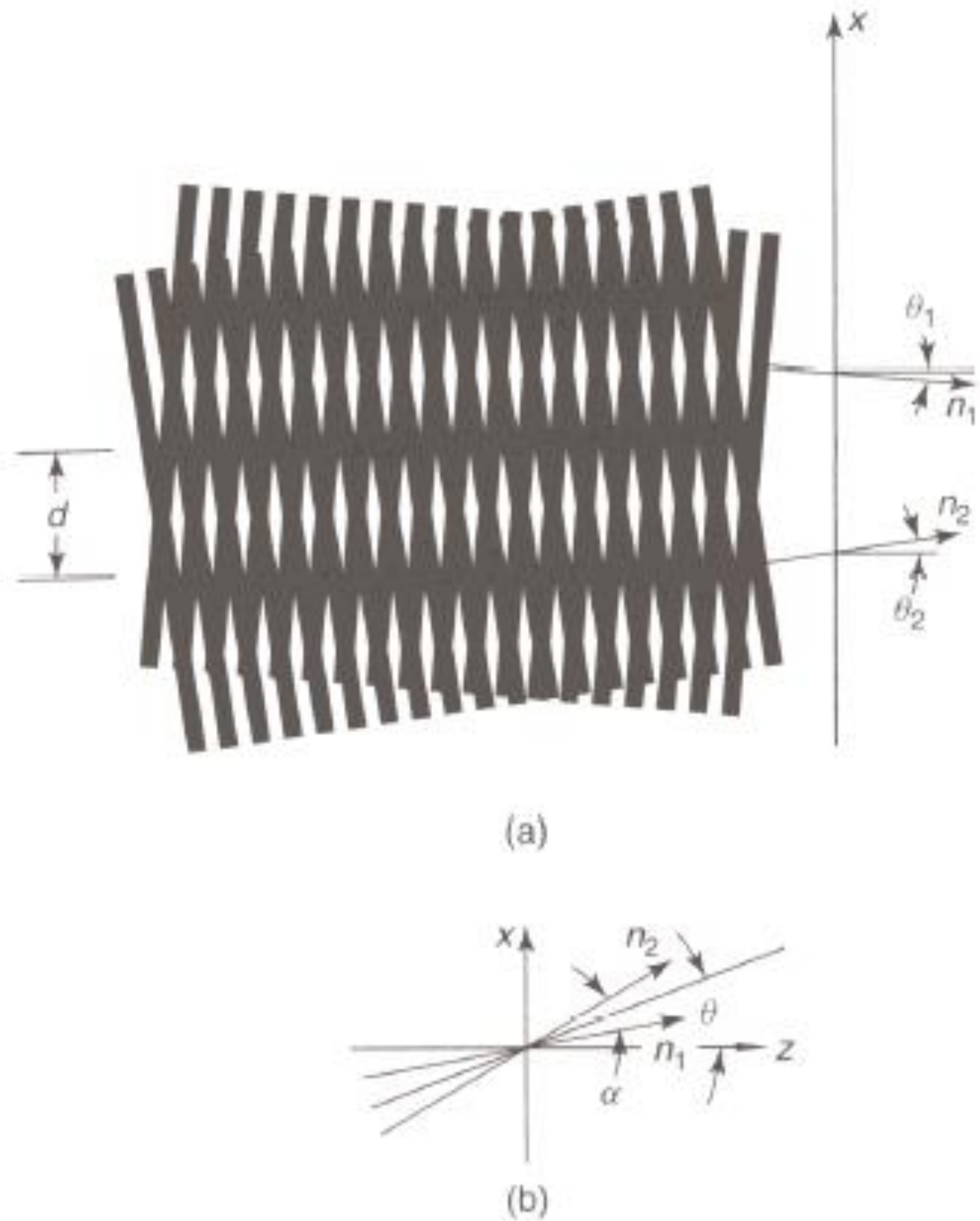
The resolving power of the grating is defined as

$$\boxed{R = \frac{\lambda}{\Delta\lambda} = \frac{\lambda_1}{\lambda_2 - \lambda_1} = mN}$$

The higher the order the better the resolution.

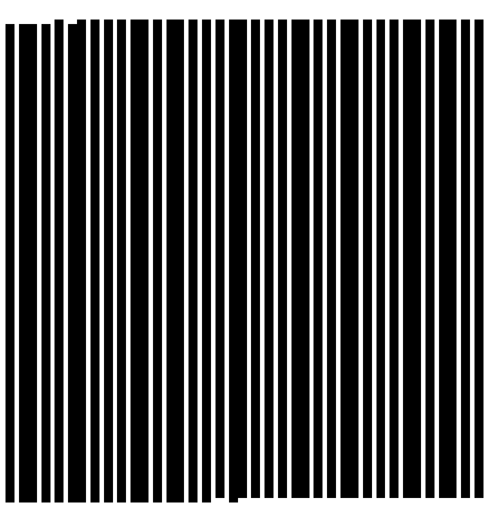
Moire Interferometry

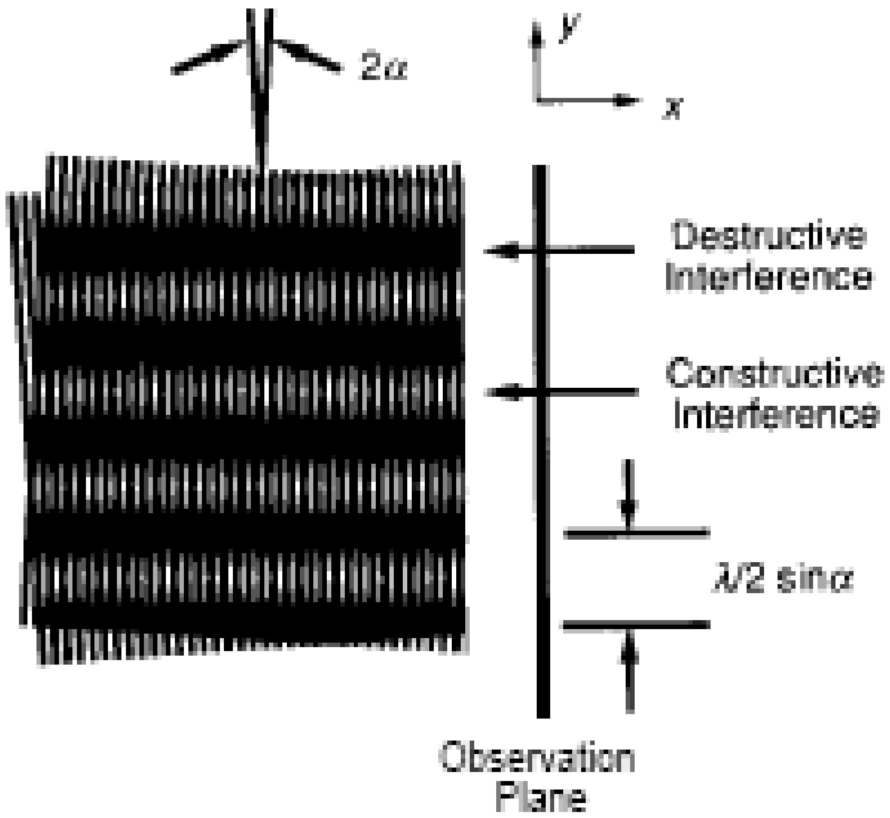




Moire Interferometry

- Dark fringe - when the dark lines are out of step one-half period
- Bright fringe - when the dark lines from one fall on the dark lines for of the other
- If the ∠ between the two gratings is increased the separation between the bright and dark fringes decreases



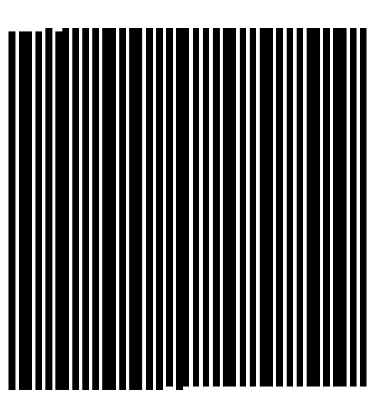


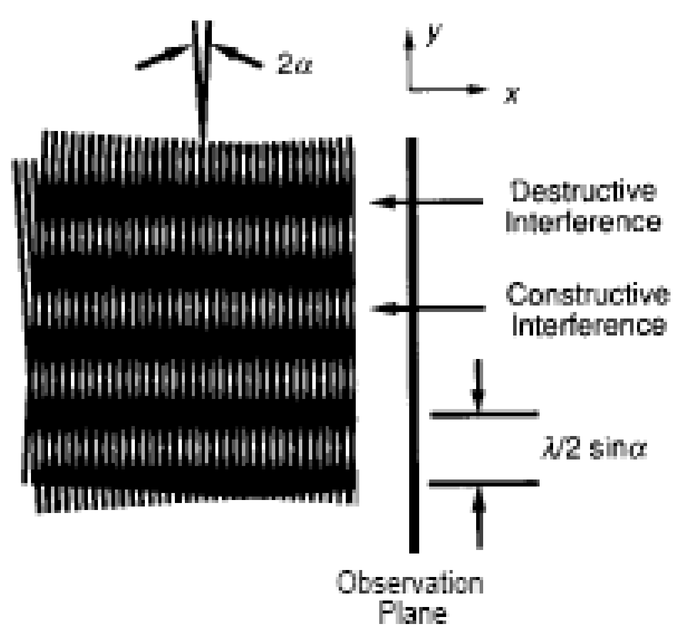
Moiré pattern formed by two line gratings rotated by small ∠

$M\lambda = 2y \sin\alpha$

Moire Interferometry

- Dark fringe - when the dark lines are out of step one-half period
- If the gratings are not identical, the moiré pattern will not be straight equi-spaced fringes $\lambda_{beat} = \dfrac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}$.
- How are moiré patterns related to interferometry?
- The grating shown in Fig. can be a "snapshot" of plane wave traveling to the right, and the grating lines distance = λ of light.





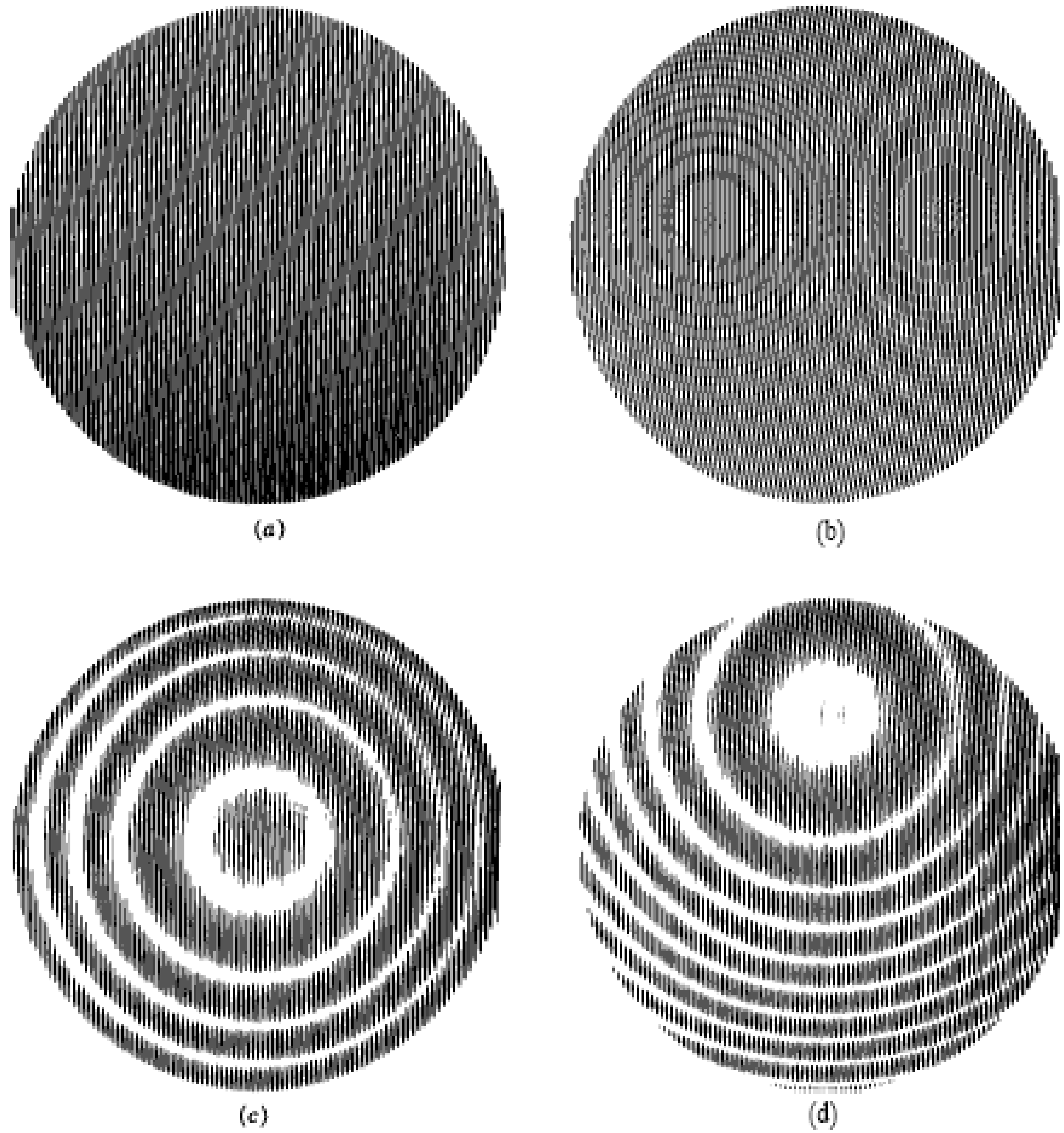
$M\lambda = 2y \sin\alpha$

Moire Interferometry $d = \dfrac{\lambda}{2\sin\alpha/2}$

- It becomes like interfereing two plane waves at an angle 2α
- Where the two waves are in phase, bright fringes result, and where they are out of phase, dark fringes result
- The spacing of the fringes on the screen is given by previous eqn. where λ is now the wavelength of light ($M\lambda = 2y\ \sin\alpha$)
- Thus, the moiré of two gratings correctly predicts the centers of the interference fringes produced by interfering 2 plane waves
- Since binary gratings are used, the moiré does not correctly predict the sinusoidal intensity profile of the interference fringes.

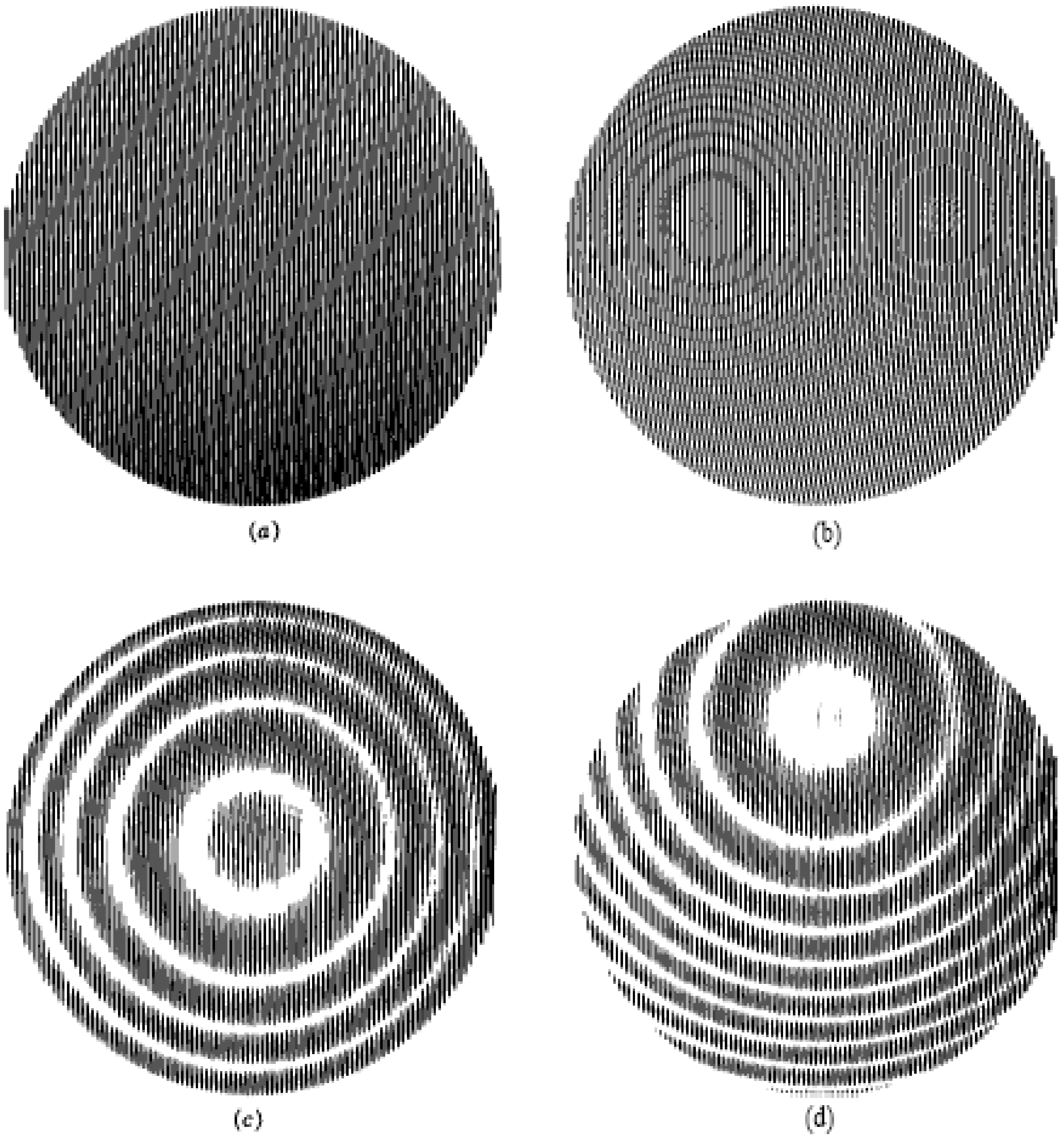
Moire Interferometry

- Fig shows the moiré produced by superimposing two computer-generated interferograms.
- First interferogram (a) has 50 waves of tilt across the radius
- Second interferogram (b) has 50 waves of tilt plus 4 waves of defocus.



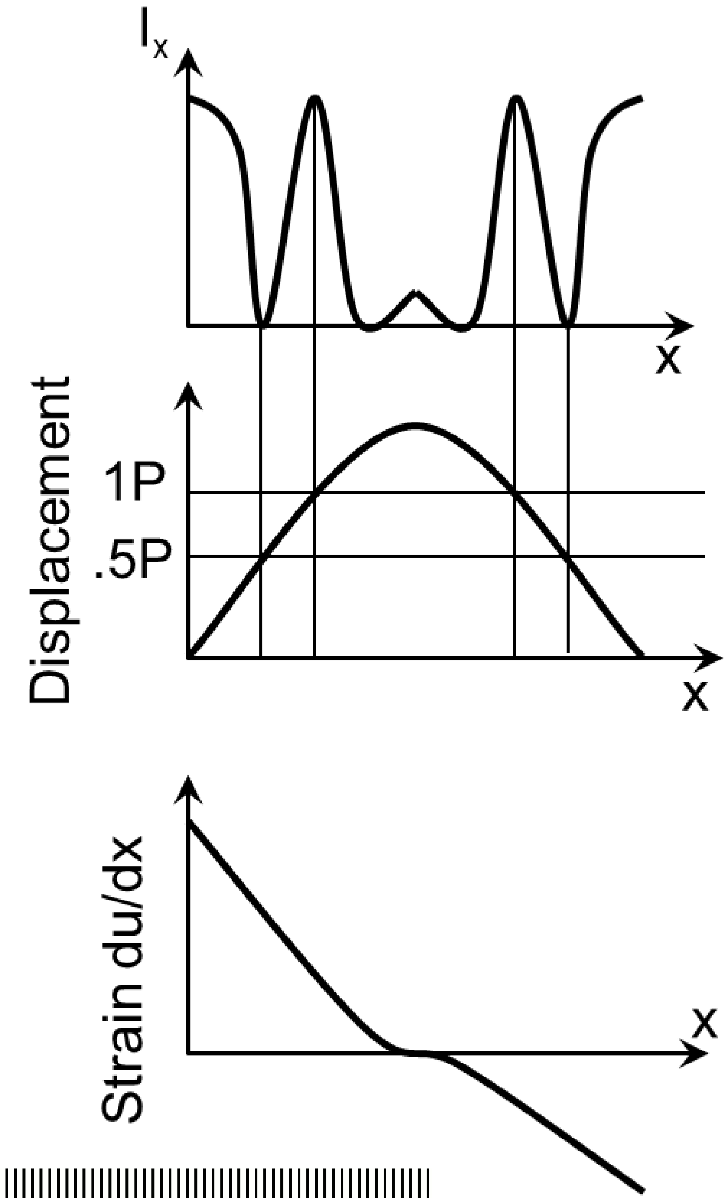
Moire Interferometry

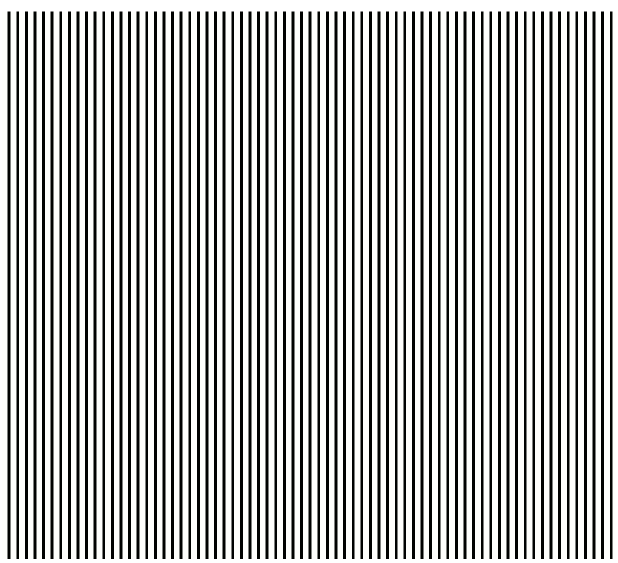
- If they are aligned such that the tilt is same for both, tilt cancels and the 4 waves of defocus remain (c).
- In (d), the two inferograms are rotated wrt each other so that the tilt will quite cancel.
- These results can be described mathematically using two grating functions:



Moire – In Plane Measurement

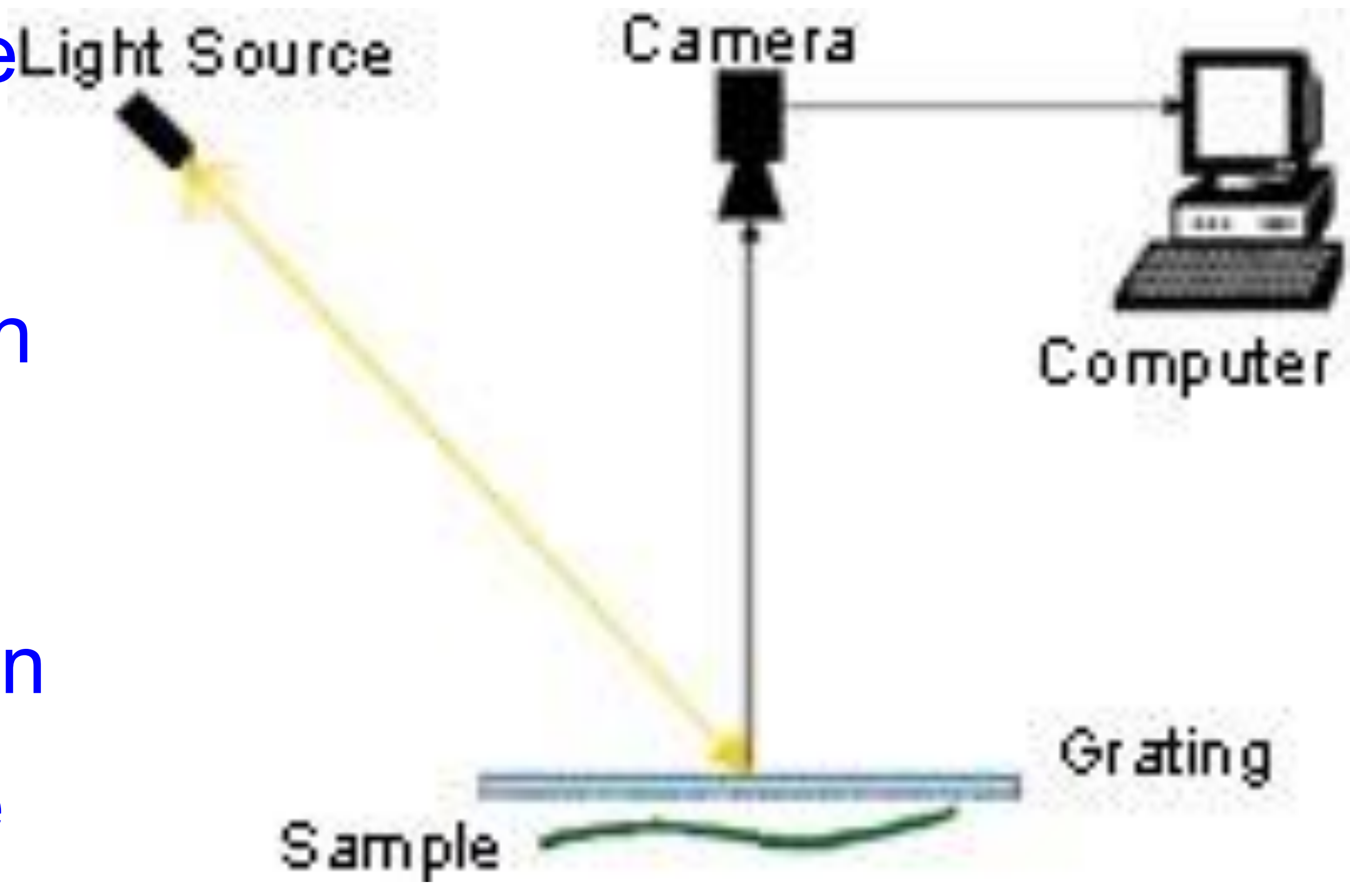
- Gratings used in Moire measurements are usually transparencies and if this is placed in contact with object, the phase of this grating will be modulated depending on the object displacement - np for maxima and (n+1/2) P for minima
- It there is a model grating as well, then the deformation produces fringes, with which the displacement can be computed
- The model grating can be placed over the grating, or imaged over the grating or imaged on photographic film





Shadow Moire – Out of Plane

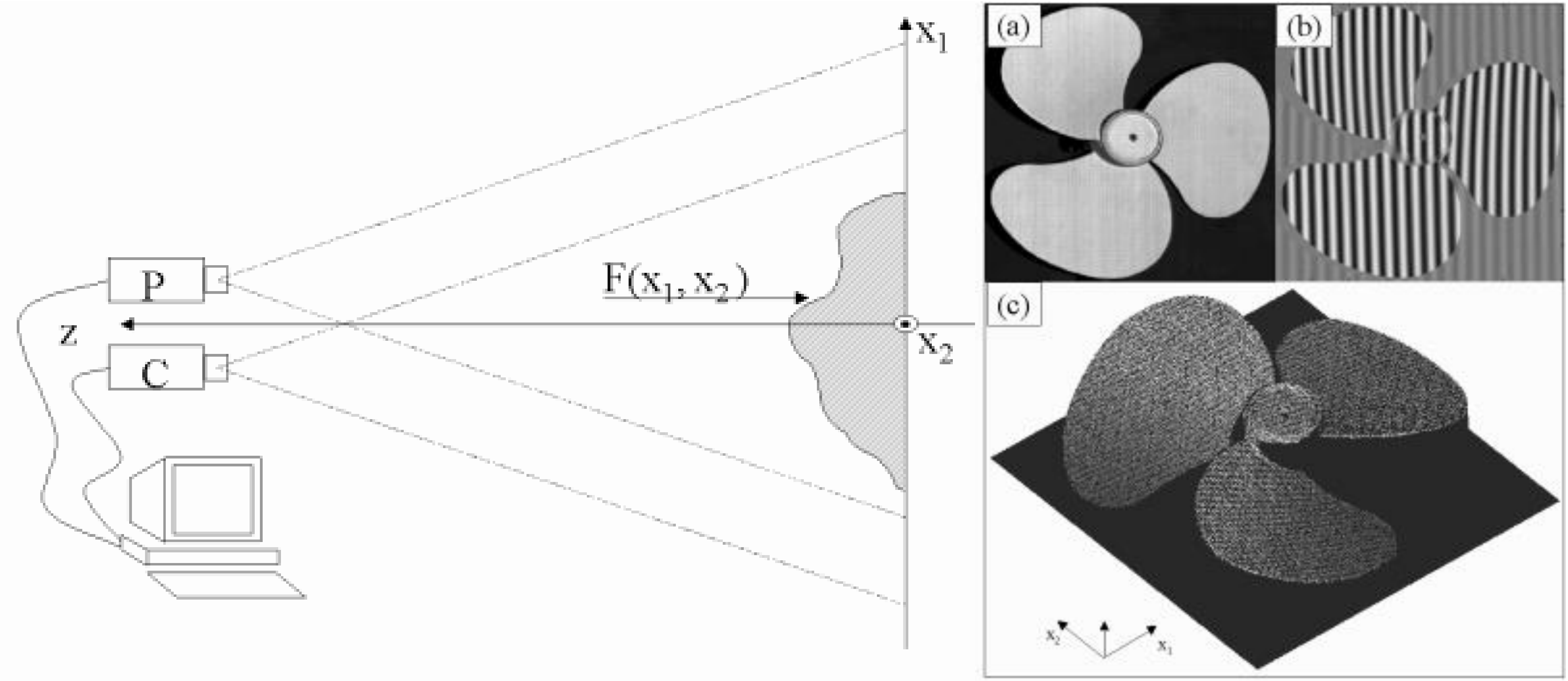
- Out of plane displacements are measured by using a single grating and an interference with the shadow of the grating itself
- The most successful application of shadow moire is in Medicine
- Useful in coarse measurements on large surfaces with complex contours



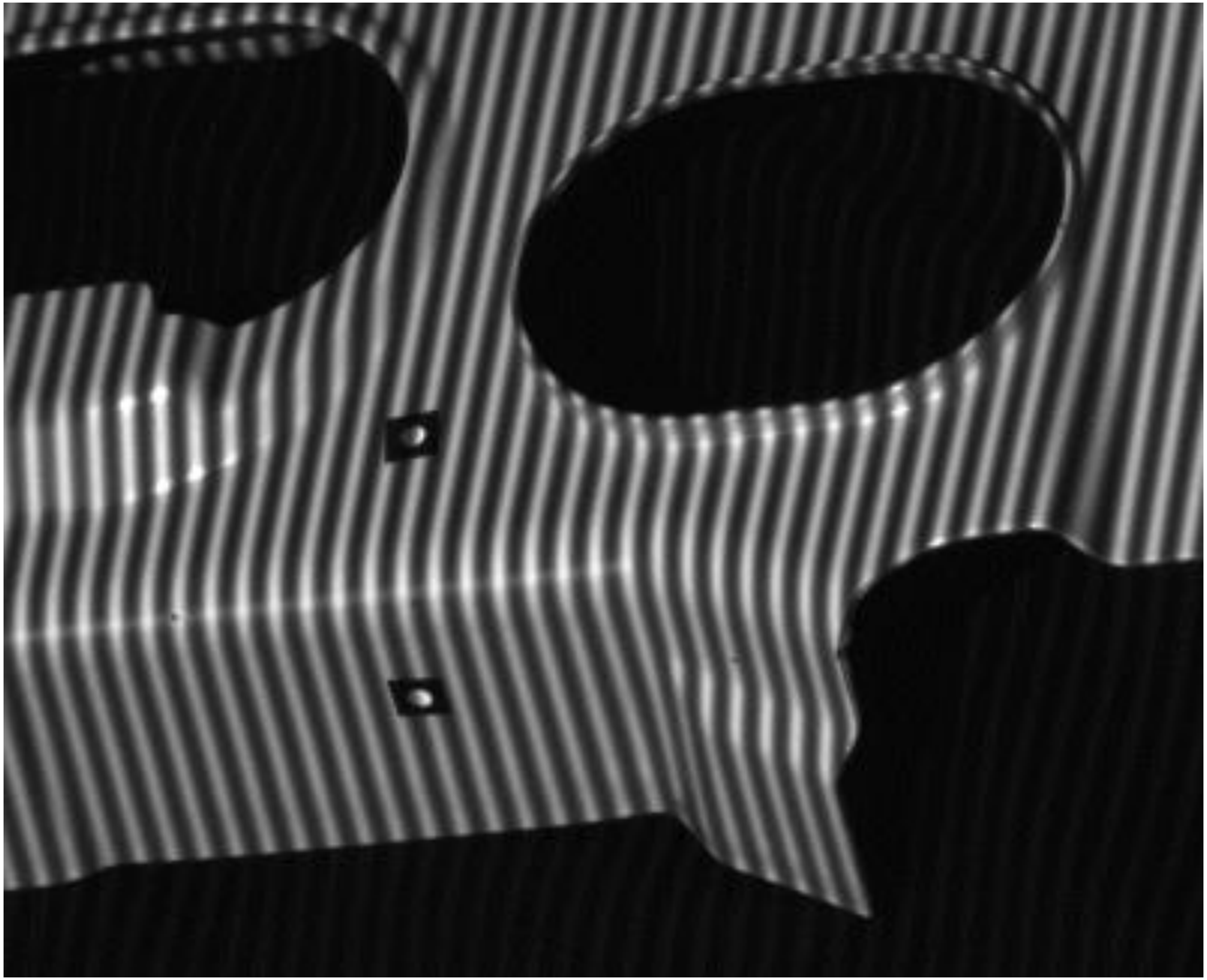
$$W = \frac{p}{\tan\alpha + \tan\beta}$$

- W = out of plane displacement
- p = grating pitch; α = light angle
- β = observation angle

Fringe Projection



Fringe Projection



Phase Shifting Interferometry

Parameters	Fringe skeletonizing	Phase stepping/ shifting	Fourier transform	Temporal heterodyning
No of interferograms to be reconstructed	1	Minimum 3	1 (2)	One per detection point
Resolution (λ)	1 to 1/10	1/10 to 1/100	1/10 1/30	1/100 to 1/1000
Evaluation between intensity extremes	No	Yes	Yes	Yes
Inherent noise suppression	Partially	Yes	No (yes)	Partially
Automatic sign detection	No	Yes	No (yes)	Yes
Necessary experimental manipulation	No	Phase shift	No (phase shift)	Frequency
Experimental effort	Low	High	Low	Extremely high
Sensitivity to external influences	Low	Moderate	Low	Extremely high
Interaction by the operator	Possible	Not possible	Possible	Not possible
Speed of evaluation	Low	High	Low	Extremely low
Cost	Moderate	High	Moderate	Very high

Comparison of phase evaluation methods

Phase Shifting Interferometry

- In phase shifting interferometers, the reference wave is shifted wrt the test wave, changing their phase difference.
- By measuring the irradiance changes for different phase shifts, it is possible to determine the phase of test wave.
- The intensity pattern of any interfering beams at any given point is governed by the following equation
- $$I = I_0[1 + V\cos\{\phi + \alpha\}]$$
- where ϕ is the phase difference distribution across the interference pattern and V is the modulation of the fringes.

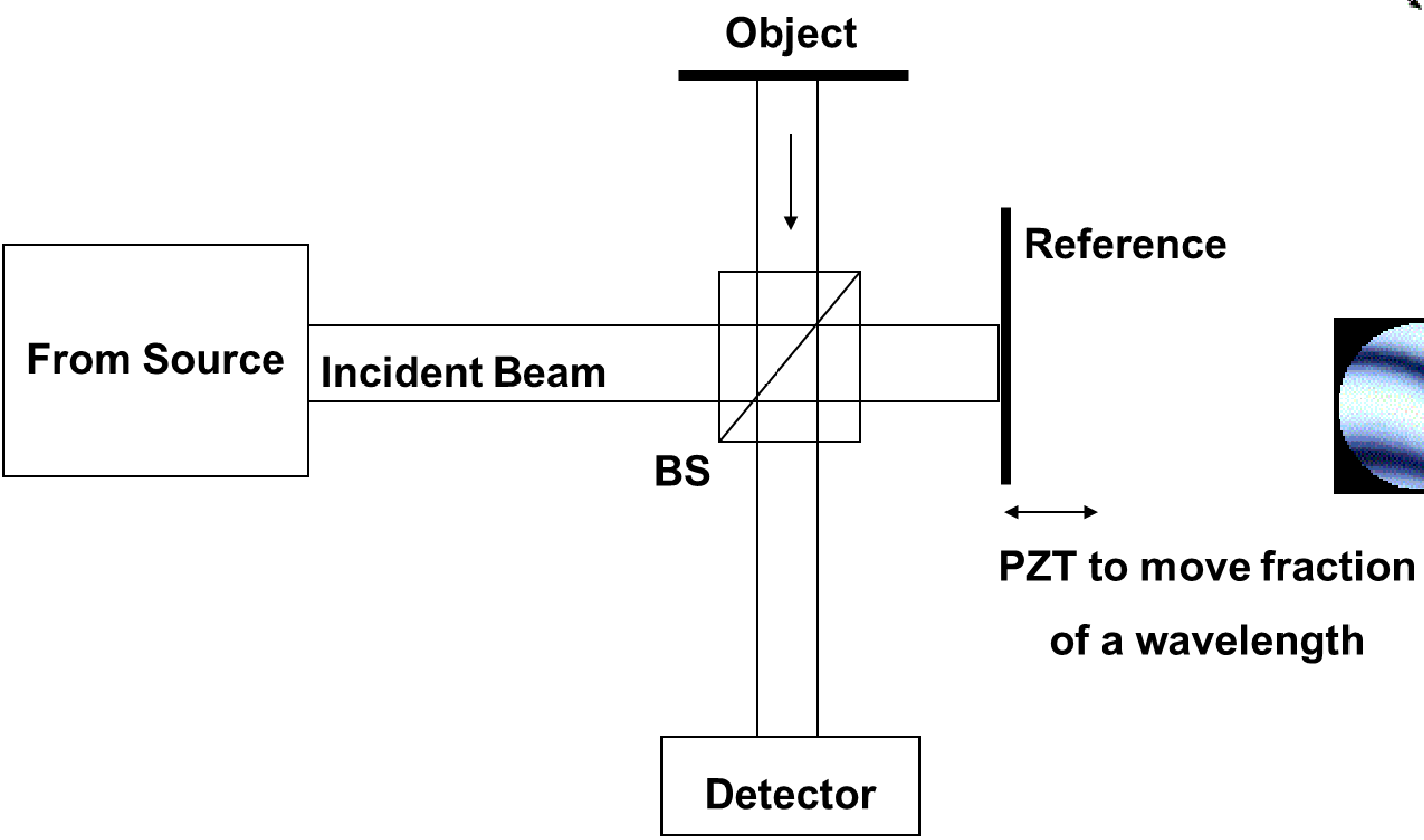
Phase Shifting Interferometry

- With CCDs, the intensity at multiple points can be recorded and processed simultaneously at high speeds. Therefore, the equation can be:
- $I\ (x, y) = I_0(x, y)[1 + V\cos\{\phi(x, y) + \alpha\}]$
- where I(x, y) is the intensity of the interference pattern at the corresponding pixel of the CCD camera, ϕ(x, y) is the phase difference at that particular pixel
- 3 unknowns in I_0, V and ϕ. Therefore, a minimum of three phase-shifted images is required to find out the phase ϕ value of a particular point

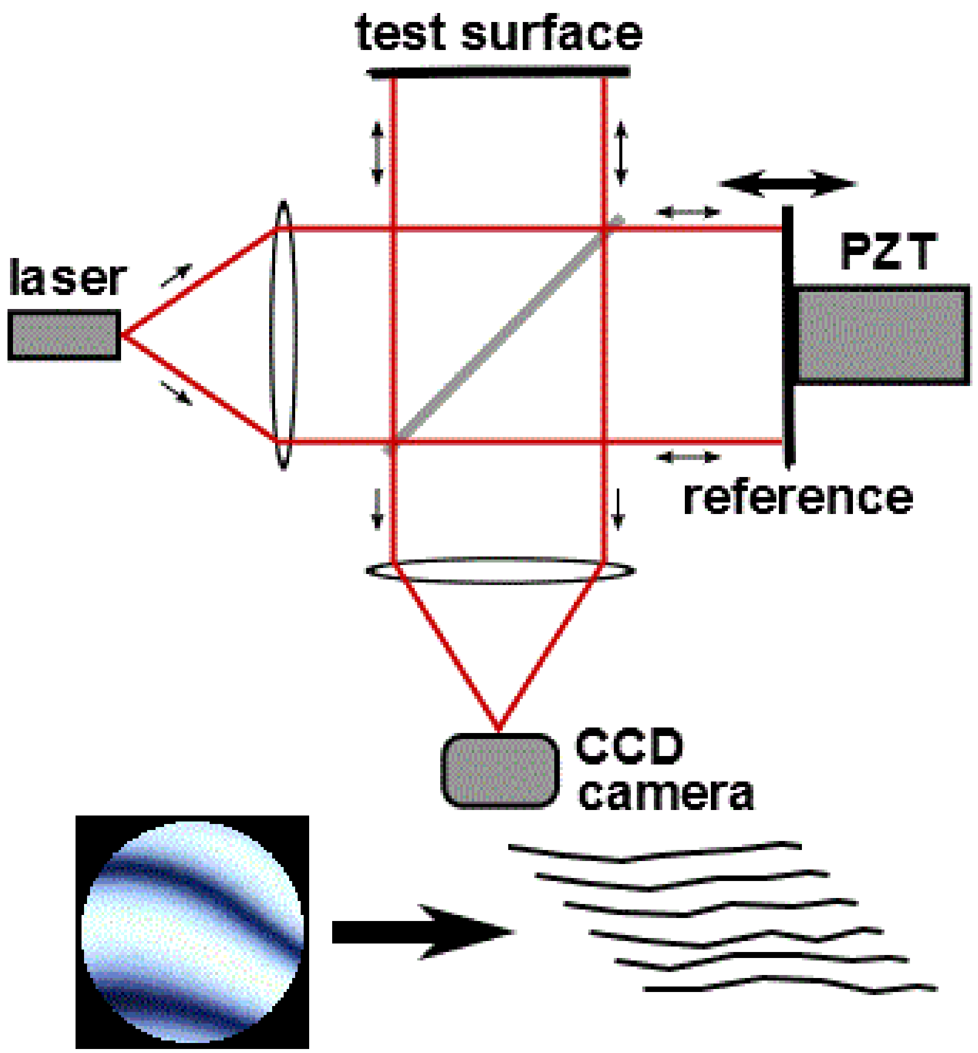
Phase Shifting Interferometry

- 2 waves derived from a common source, the phase difference between the two waves is given by $\Delta p = \frac{\lambda}{2\pi} \Delta \phi$
- where $\Delta\phi$ is the phase difference, ΔP is the path difference between them and, λ the wavelength.
- The phase difference could be introduced by introducing path difference and vice versa
- In most of the phase shifting interferometric techniques, changing the path length of either the measurement, or the reference beam, by a fraction of the wavelength provides the required phase shifts

Phase Shifting Interferometry



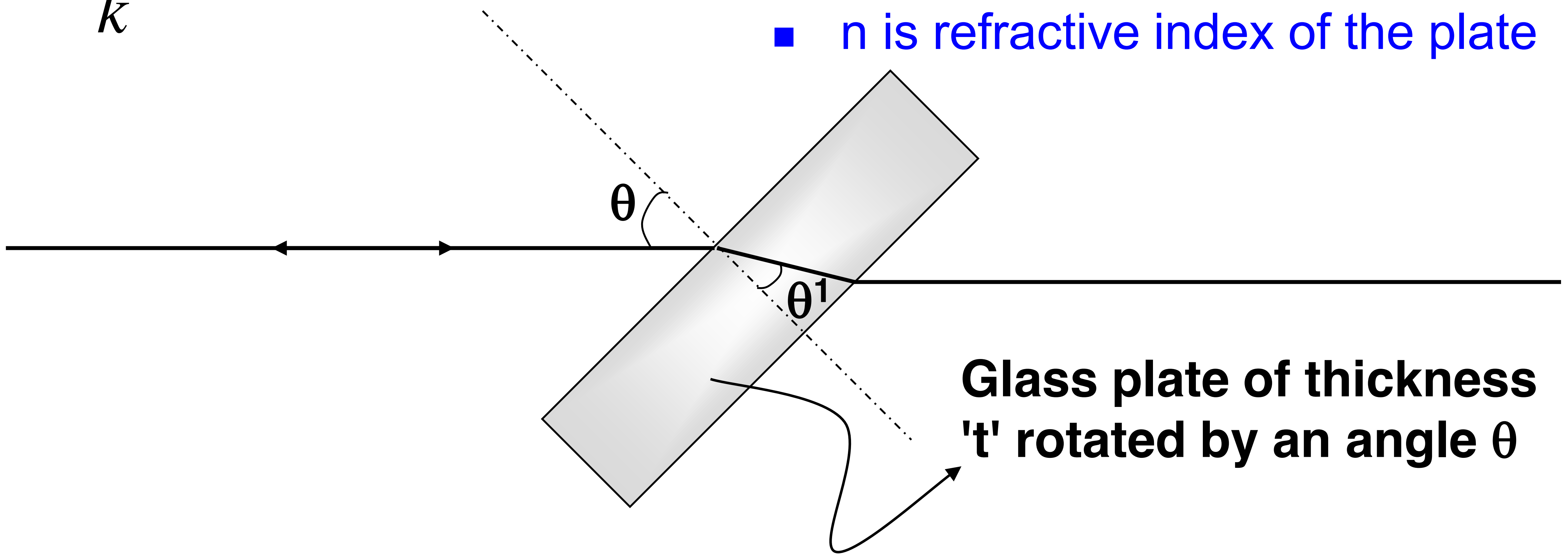
Phase shifting technique (Piezo Mirror)



Phase Shifting Interferometry

$$\alpha = \frac{t}{k}(n\cos(\theta^1) - \cos(\theta))$$

- K =2π/λ
- n is refractive index of the plate



Glass plate of thickness 't' rotated by an angle θ

Phase shifting with rotating glass plate.

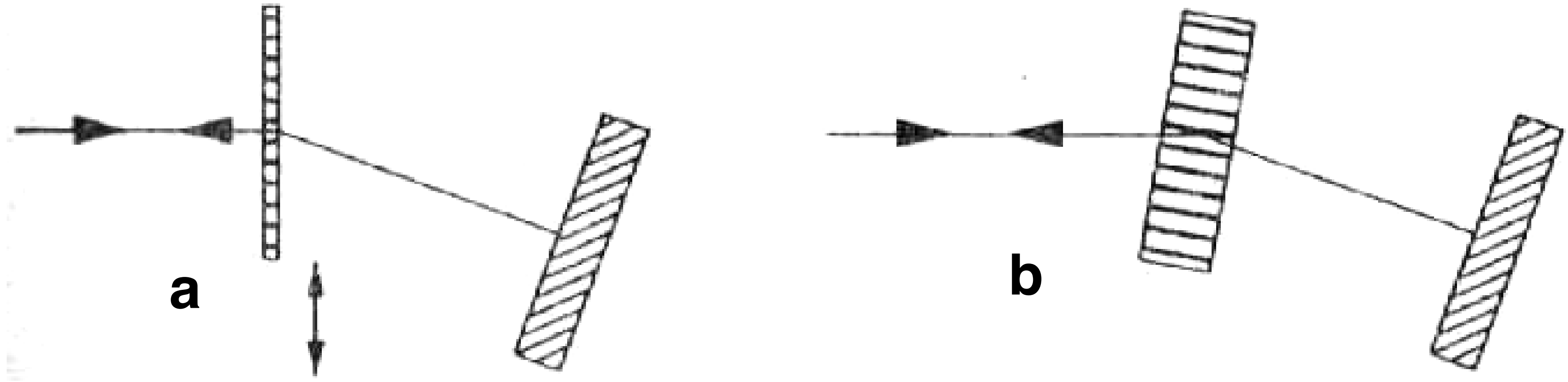
$$\alpha = \frac{t}{k}\left(1 - \frac{1}{n}\frac{\cos(\theta)}{\cos(\theta^1)}\right)\sin(\theta)\Delta\theta$$

- Phase shift achieved by rotating the glass plate by small angle

Phase Shifting Interferometry

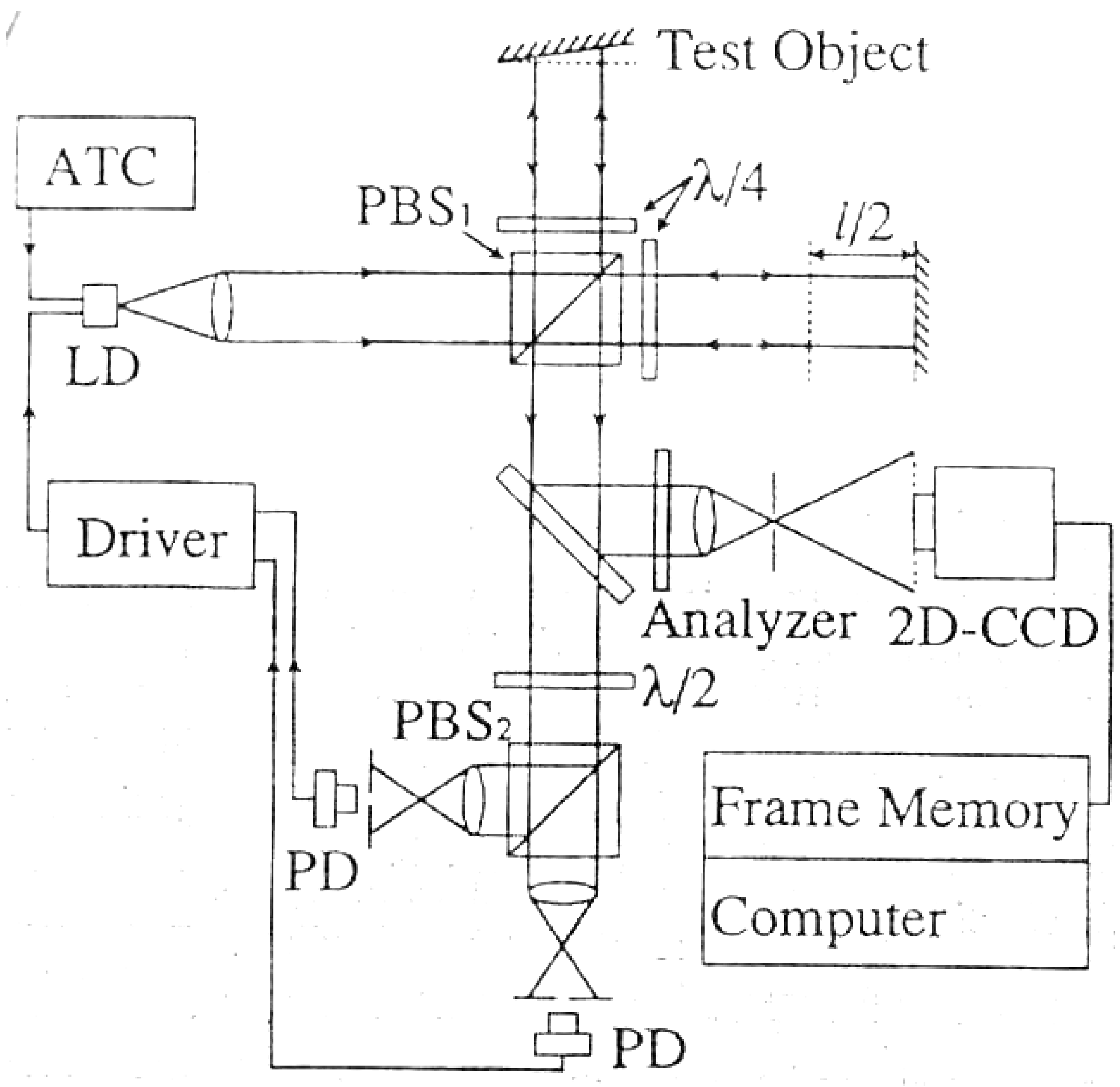
$$\alpha = \frac{2\pi n}{d}\Delta y$$

- Where d is the period of the grating and n is the order of diffraction



Phase shifting by moving grating and Bragg cell.

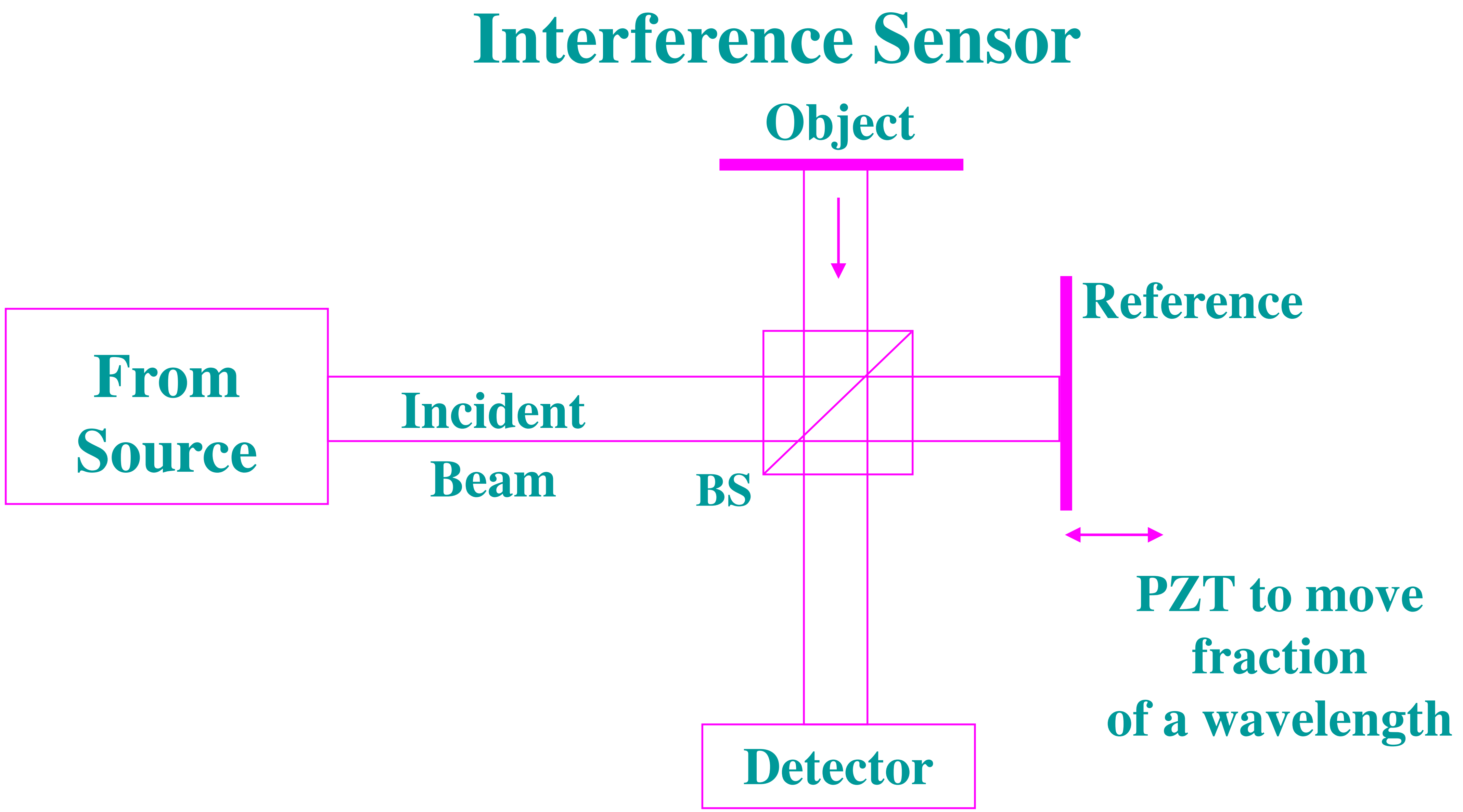
Phase Shifting Interferometry



Phase shifting with laser feedback

- the frequency of the source is changed by injecting electrical current to the laser and using a large optical path difference between the measurement and reference beams

Phase Shifting Interferometry



- ΔP is the path difference between the two beams
- $\Delta\phi$ is the phase difference between them
- λ the wavelength of the light source

Phase Shifting Interferometry

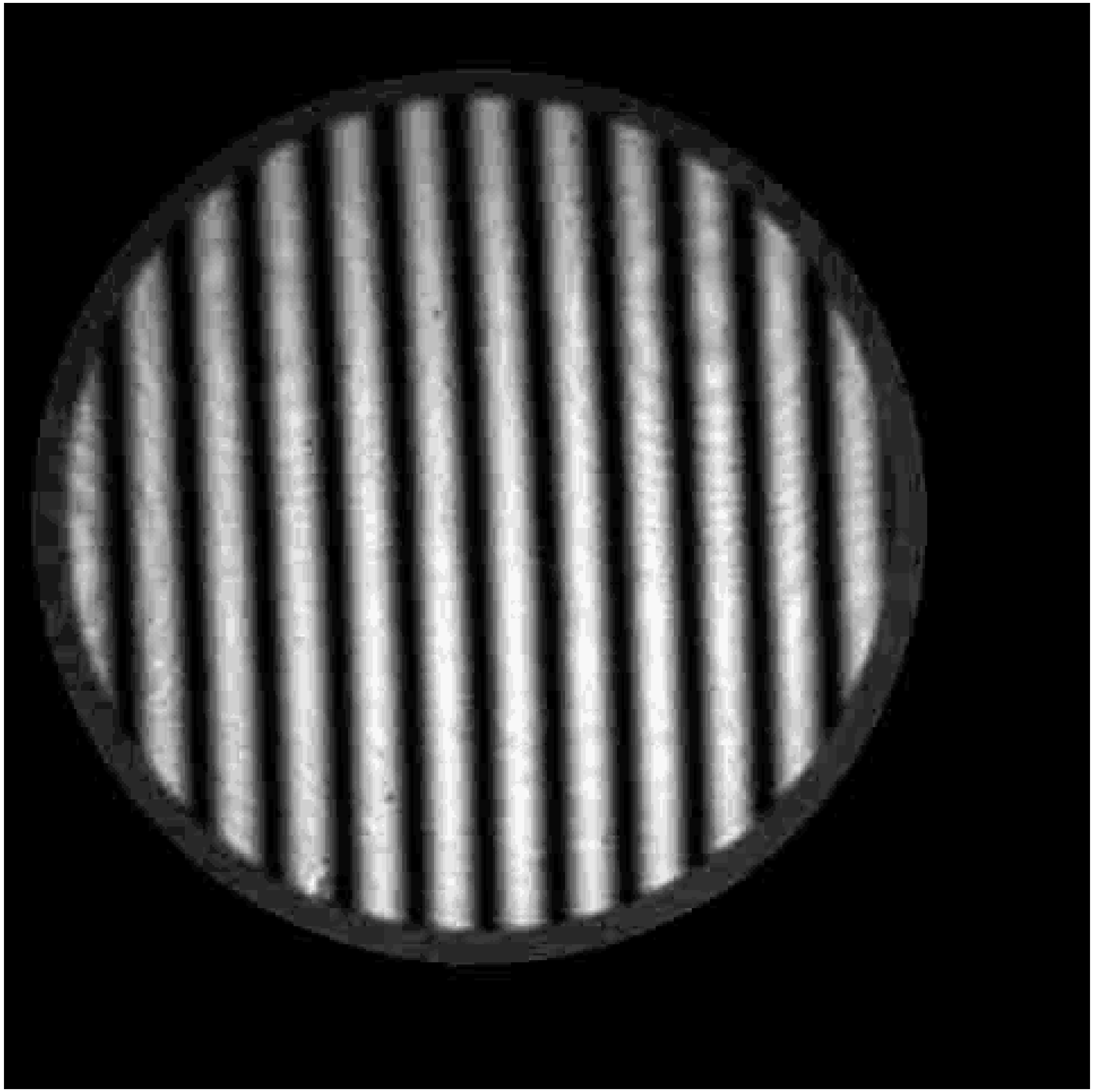
$$I_1(x, y) = I_0(x, y)\left(1 + V \cos \delta\right)$$

'I(x, y)' is the intensity of the interference pattern,

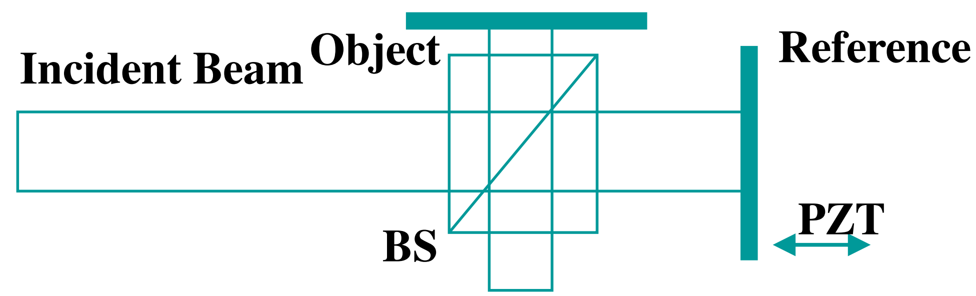
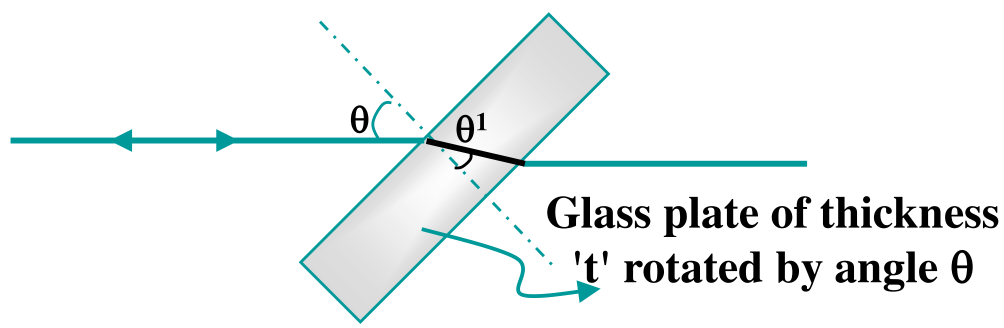
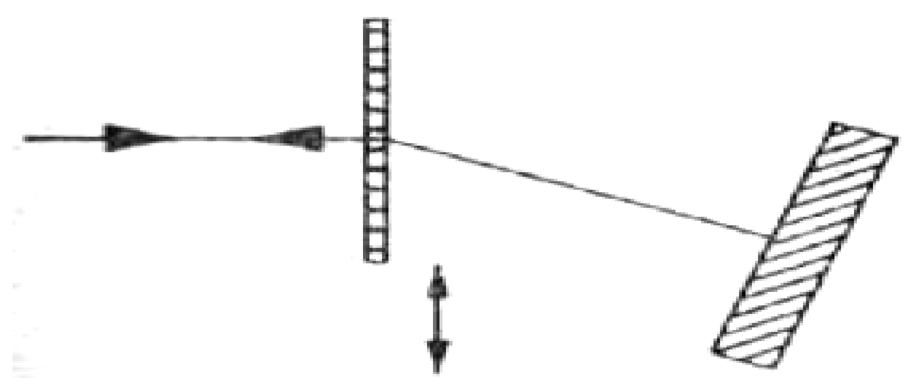
'δ(x, y)' is the phase difference between object and reference,

'V' is the modulation of the fringes

Phase Shifting Interferometry



Phase Shifting Techniques

<table>
<tr>
<td>Mirror with linear transducer</td>
<td>
</td>
<td>$$\alpha = \frac{2\pi}{\lambda}\Delta p$$
Where λ is wavelength
Δp is the path difference</td>
</tr>
<tr>
<td>Rotating glass plate</td>
<td>
</td>
<td>$$\alpha = \frac{t}{k}\left(1-\frac{1}{n}\frac{\cos(\theta)}{\cos(\theta^{1})}\right)\sin(\theta)\Delta\theta$$
Where K =2π/λ and
'n' is the refractive index</td>
</tr>
<tr>
<td>Moving diffraction grating</td>
<td></td>
<td>$$\alpha = \frac{2\pi n}{d}\Delta y$$
Where d =grating period
'n' is the refractive index</td>
</tr>
<tr>
<td colspan="2">Laser feedback</td>
<td>Where 'phase shift' is proportional to path difference and 'temporal frequency'</td>
</tr>
<tr>
<td colspan="2">Polarization based phase shifting</td>
<td>$$\alpha = 2\Delta\theta$$
Where α = phase shift
Δθ = angle of rotation</td>
</tr>
</table>

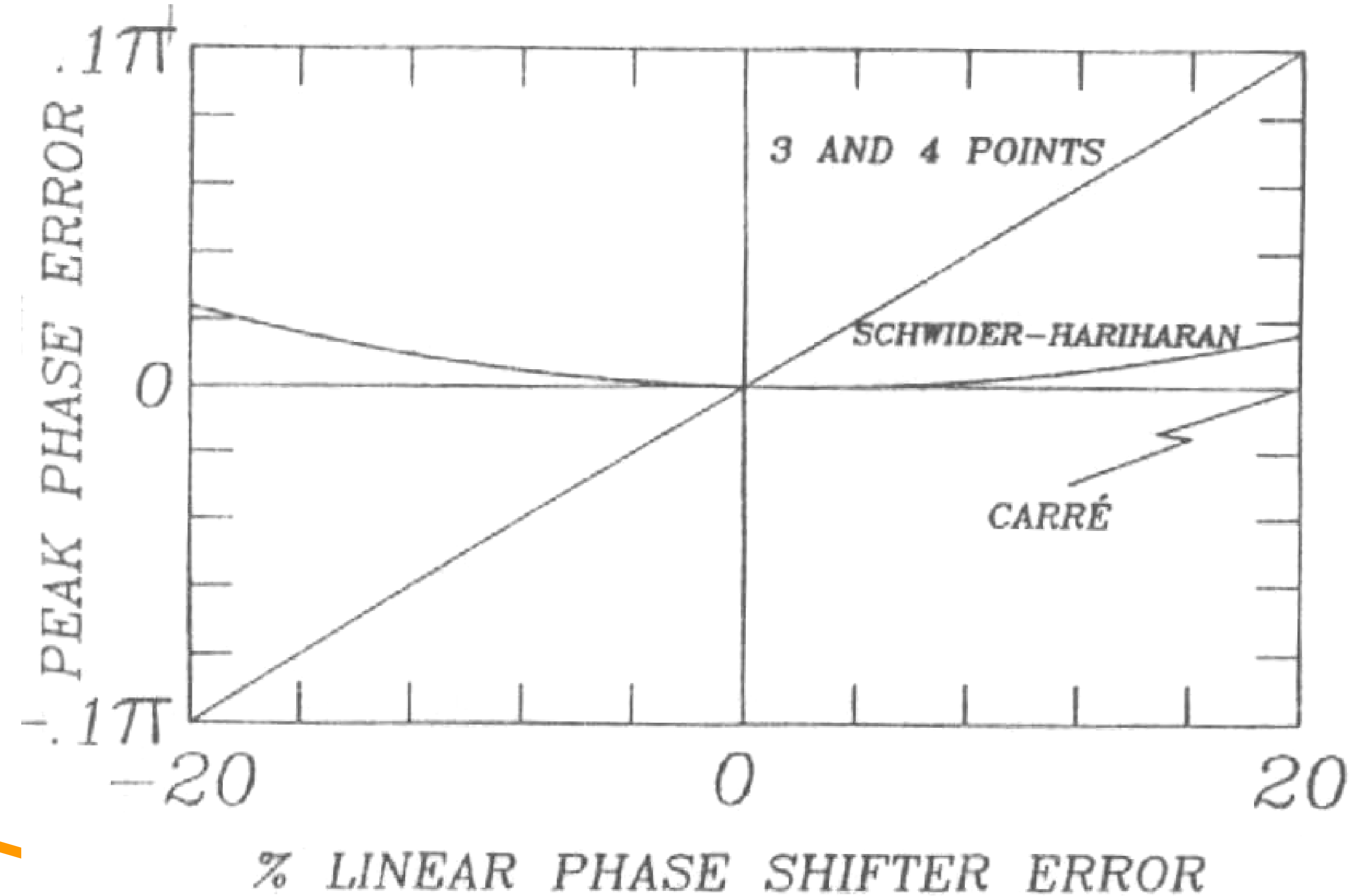
Phase Shifting Algorithms

Three step method	$Tan\delta = \frac{\sqrt{3(I_1 - I_3)}}{2I_2 - I_1 - I_3}$ $\quad Tan\delta = \frac{I_2 - I_1}{I_3 - I_2}$
Four step method	$Tan\delta = \frac{I_4 - I_2}{I_1 - I_3}$
Carré method	$Tan\delta = \frac{\sqrt{[3(I_2 - I_3) - (I_1 - I_4)][(I_2 - I_3) + (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)}$
Five step method	$Tan\delta = \frac{2(I_2 - I_4)}{2I_3 - I_5 - I_1}$

Other algorithms like ‘Integrated Bucket Technique’ for continuous phase shifting and ‘multi-step techniques’ have been used

Phase Shifting Errors

- Linear phase error
- Non linear phase error
- Light detector anomalies
- Vibration & air turbulence
- Other random errors



All these are errors associated with mechanical phase shifters