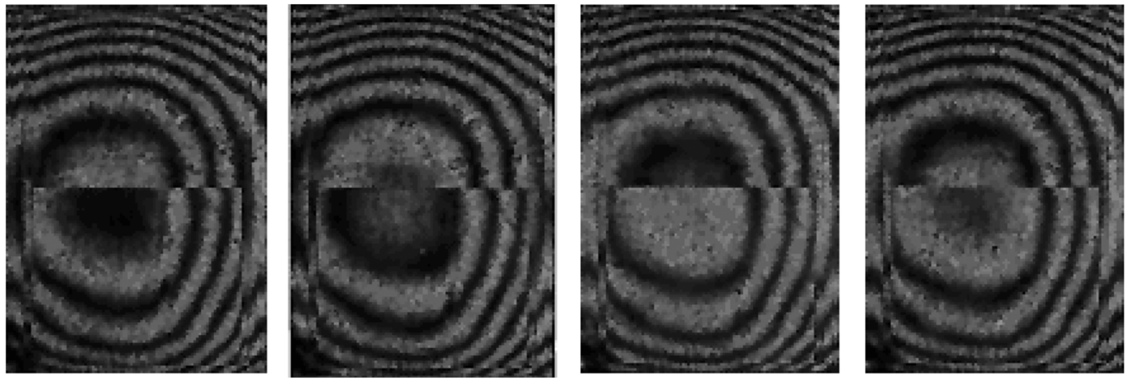
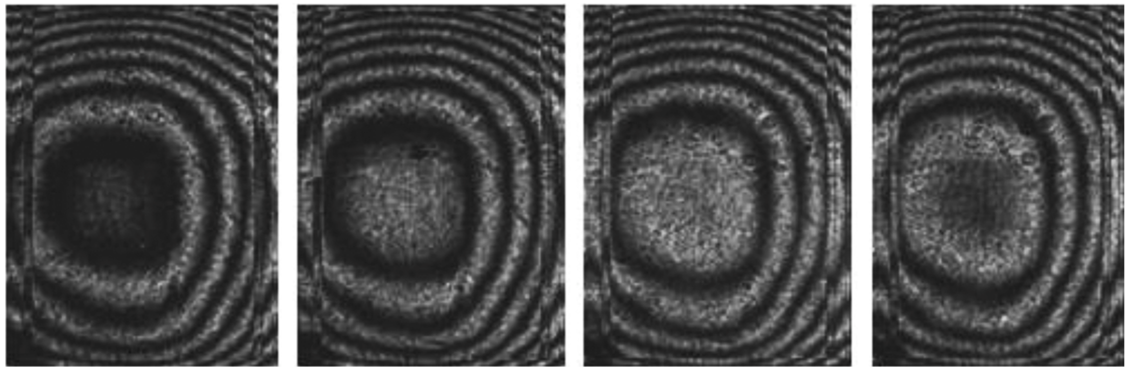
Optical Metrology

Lecture 6: Phase-Shifting Systems and Phase-Shifting Analysis

Introduction

- Phase-shifting surface profile measurement is an important branch in optical metrology.
- It has many unique features such as varieties of configurations, high resolution, high accuracy, good repeatability, fast measurement speed, and superior surface finish tolerance.
- Digital image devices have enabled automated processing of phase-shifting images at high speed over a full field of view (FOV), further enabling superfast 3D measurement without scanning.


(a)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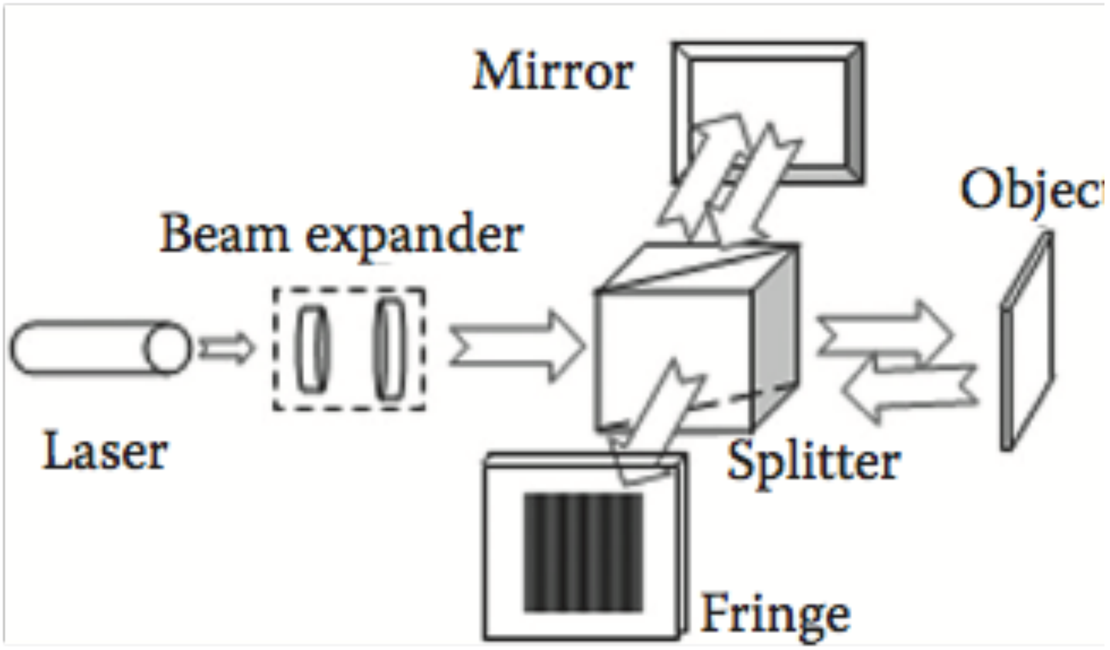
Fringe Pattern: Interference

A fringe pattern is a periodic grayscale pattern with alternative dark and bright areas.

Based on pattern generation principles, the most common fringe pattern can be classified into three categories: interference pattern, moiré pattern, and projected pattern.



(a)

Michelson Interferometer

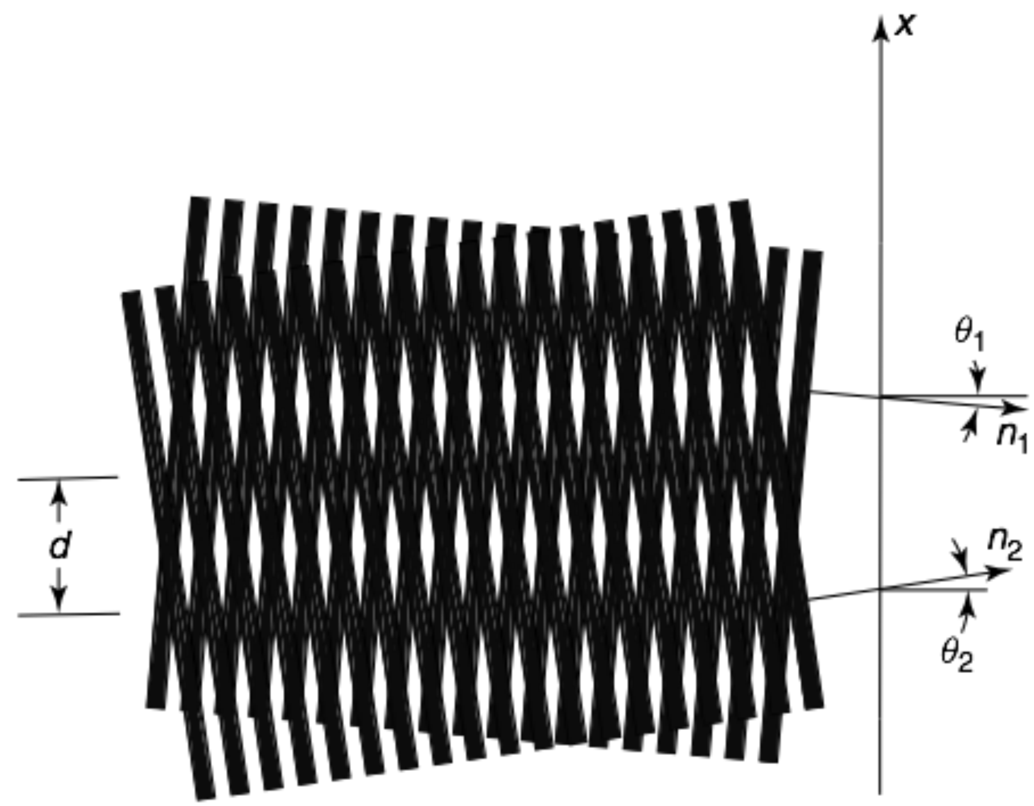
Fringe Pattern: Moiré

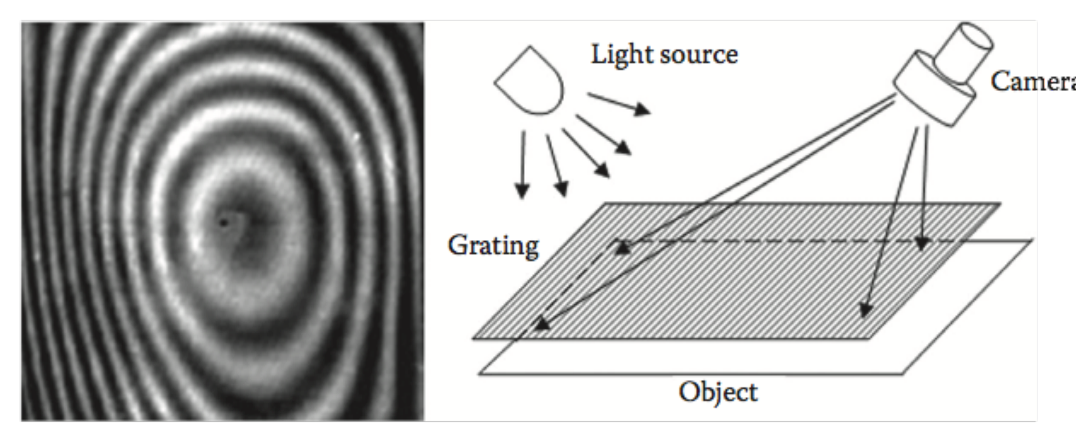
Looks like an interference pattern, but geometric interference principle is different.

It is generated by covering the measurement area with a physical grating while illuminating and viewing the area from an opposite direction.

When light passes through the grating at a tilted angle, a shadow of the grating will be generated on the sample surface.

When this shadow is observed at a tilted angle from the opposite direction to the light source, a moiré pattern can be seen that represents the topology of the surface with the peak/valley rings representing the same height relative to the physical grating.





(a) (b)

FIGURE 7.2
Shadow moiré: (a) moiré pattern and (b) setup.

Fringe Pattern: Projection with physical grating

- When a transmission grating with a sinusoidal transmission profile such as holographic gratings is placed between a light source and a projection lens, the projected fringe pattern will also have a sinusoidal intensity profile.

- If a straight-line grating with a nonsinusoidal profile such as a ruled grating is used, the projection lens is usually defocused slightly so that a pseudo-sinusoidal pattern can be obtained

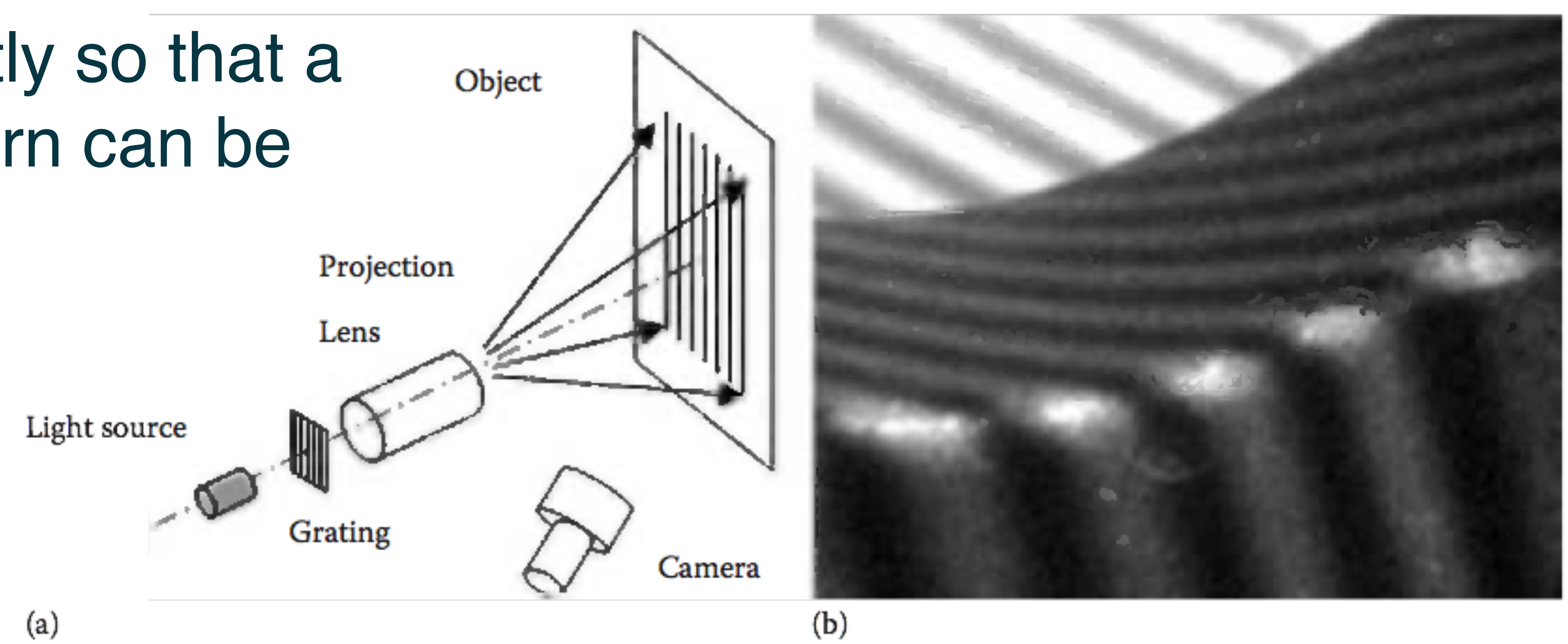


FIGURE 7.3
Projection moiré: (a) setup and (b) pattern.

Fringe Pattern: Digital Fringe Projection

- The fringe pattern can be generated with theoretically any intensity profile using computer software and projected to the object surface through an off-the-shelf digital projector such as liquid crystal device (LCD), digital mirror device (DMD), and liquid crystal on silicon (LCOS) projectors.

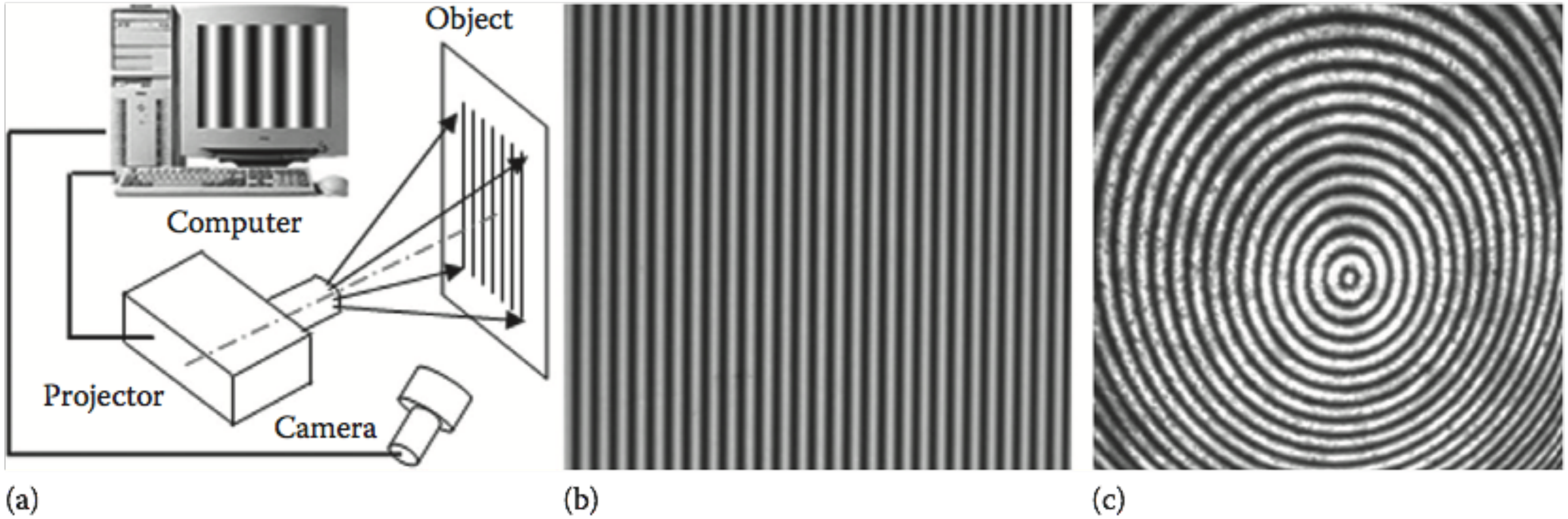


FIGURE 7.4
Digital fringe projection: (a) setup, (b) straight fringe pattern, and (c) circular fringe pattern.

Fringe Pattern Analysis

Contour Analysis

Before phase-shifting, the only way to investigate the fringe pattern image was to count the peak and/or valley and follow the contour curve along the peaks and valleys.
The calibration process was to find the factor that converted the peak/valley into the height dimension and was used to estimate the height variation over the FOV.
The resolution and accuracy of this analysis method was very low, and there is no way to identify the direction of the slopes from a single image.

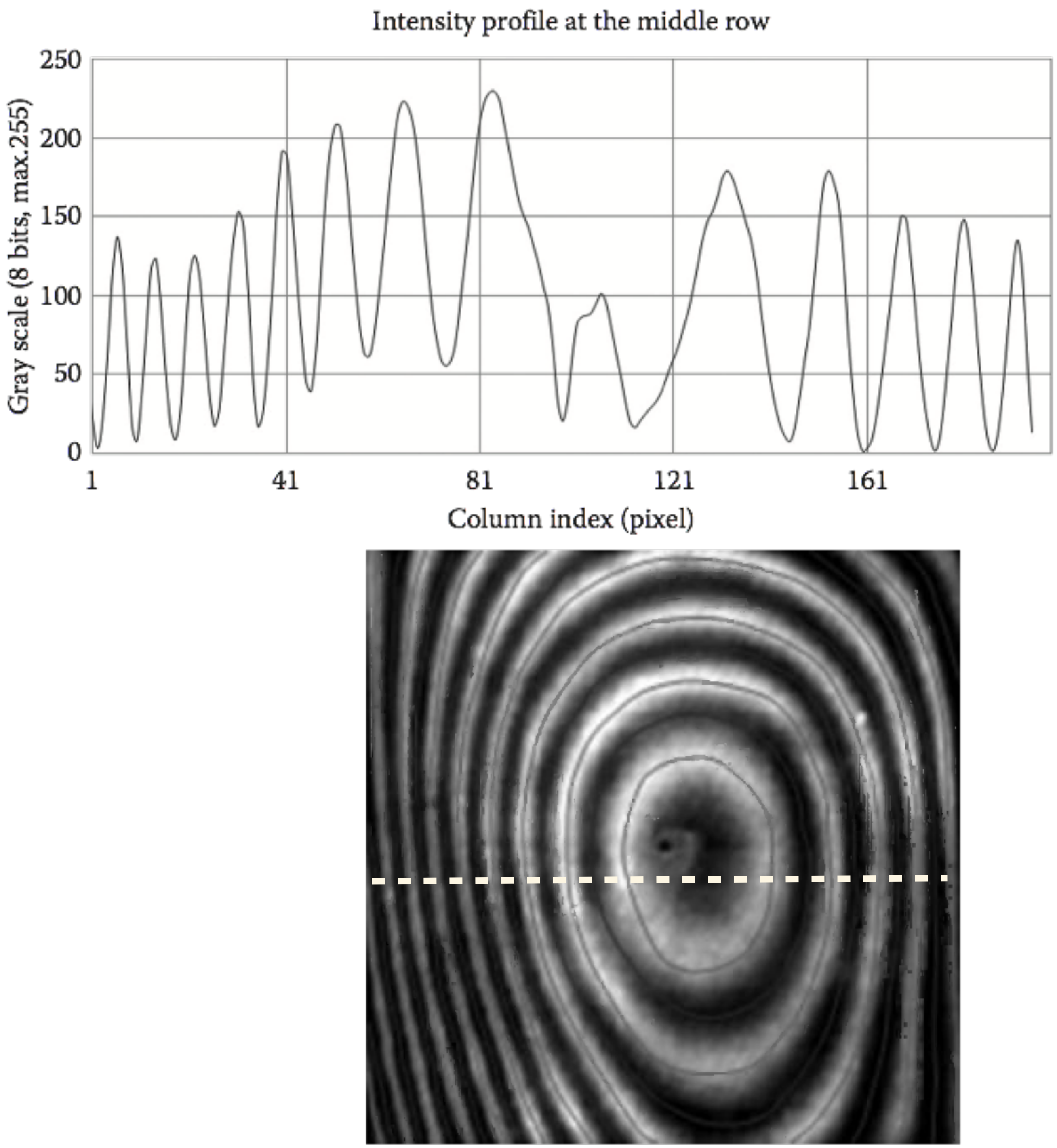
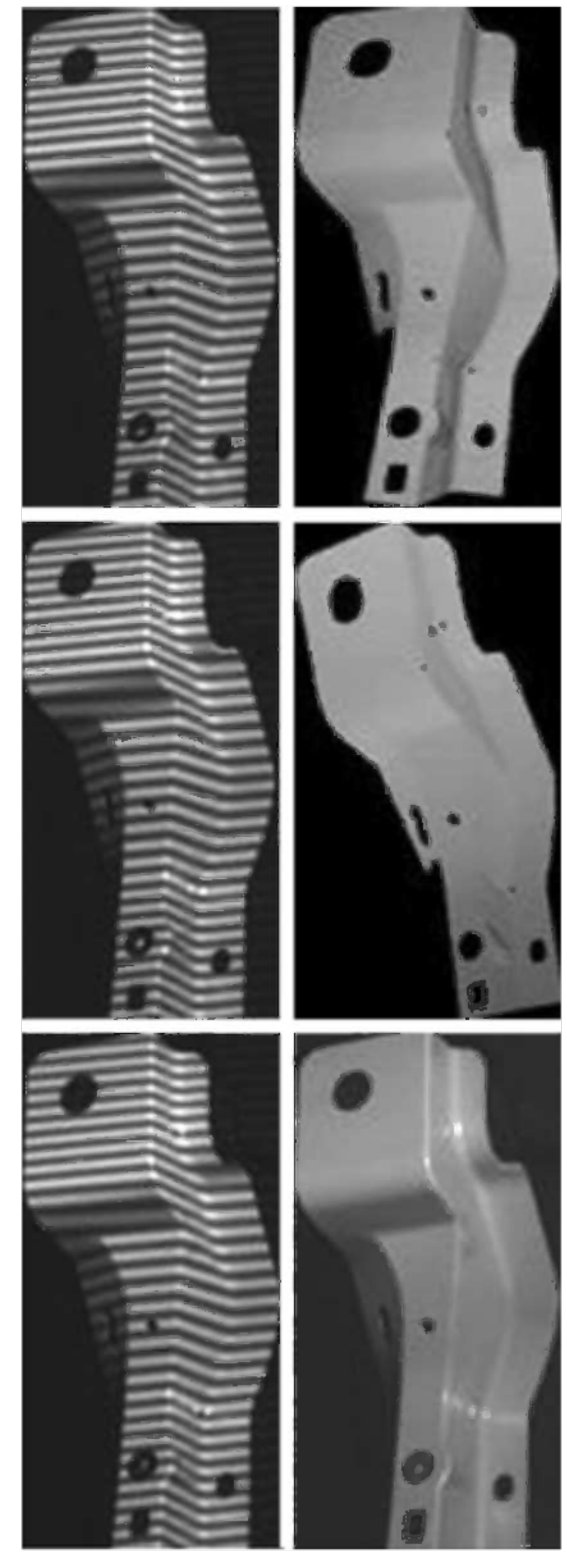


FIGURE 7.6
Contour showing peak (bright bands) and valley (dark bands).

Phase-Shifting Analysis



- An entire phase-shifting analysis process is demonstrated in the figure using a three-step phase-shifting algorithm.
- The three images in the left column are captured three fringe images of a master model with a 120° phase shift. The intensities of the three phase-shifted images at point (x, y) can be written as:

$$I_1(x,y) = I'(x,y) + I''(x,y)\cos\left[\phi(x,y) - \frac{2\pi}{3}\right] = I'(x,y)\left\{1 + \gamma(x,y)\cos\left[\phi(x,y) - \frac{2\pi}{3}\right]\right\} \tag{7.1}$$

$$I_2(x,y) = I'(x,y) + I''(x,y)\cos\left[\phi(x,y)\right] = I'(x,y)\left\{1 + \gamma(x,y)\cos\left[\phi(x,y)\right]\right\} \tag{7.2}$$

$$I_3(x,y) = I'(x,y) + I''(x,y)\cos\left[\phi(x,y) + \frac{2\pi}{3}\right] = I'(x,y)\left\{1 + \gamma(x,y)\cos\left[\phi(x,y) + \frac{2\pi}{3}\right]\right\} \tag{7.3}$$

where

$I'(x, y)$ is the average intensity

$I''(x, y)$ is the intensity modulation

$\phi(x, y)$ is the phase to be determined

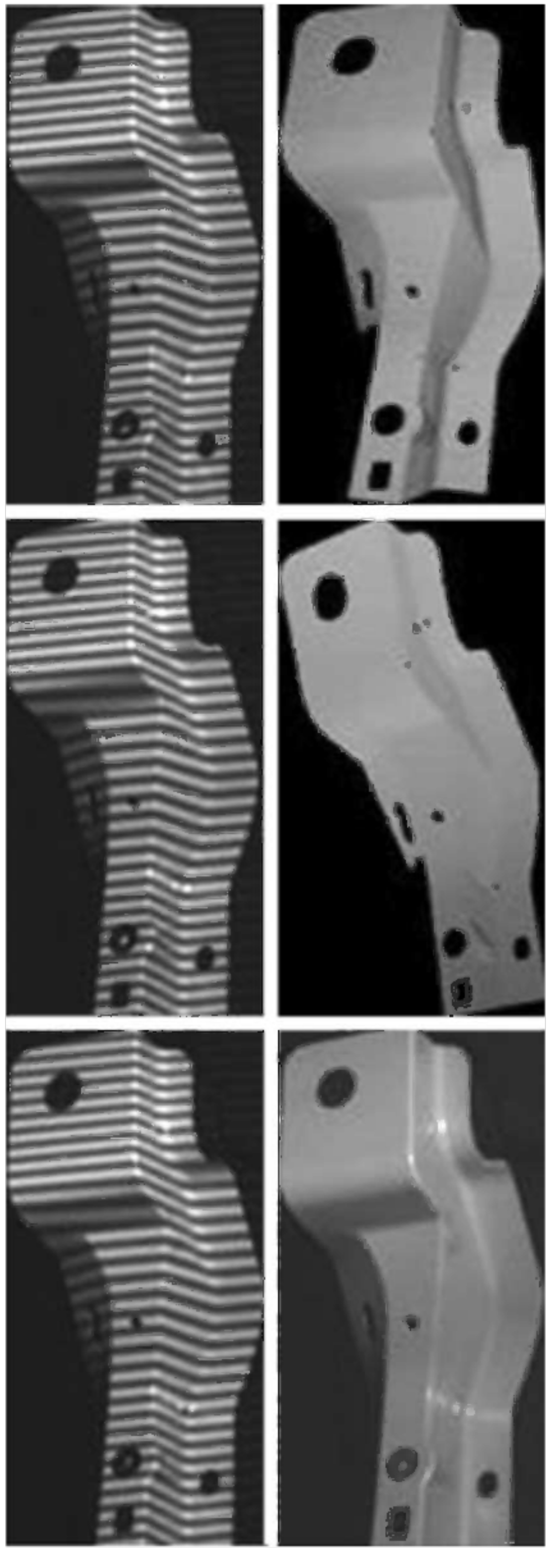
Phase-Shifting Analysis

- An entire phase-shifting analysis process is demonstrated in the figure using a three-step phase-shifting algorithm.
- The three images in the left column are captured three fringe images of a master model with a 120° phase shift. The intensities of the three phase-shifted images at point (x, y) can be written as:

By solving the earlier equations, phase $\phi(x, y)$ and image contrast $\gamma(x, y)$ can be obtained as

$$\phi(i, j) = \tan^{-1}\left(\sqrt{3}\,\frac{I_1 - I_3}{2I_2 - I_1 - I_3}\right) \tag{7.4}$$

$$\gamma(i, j) = \frac{I''(x, y)}{I'(x, y)} = \frac{\left[(I_3 - I_2)^2 + (2I_1 - I_2 - I_3)^2\right]^{1/2}}{I_2 + I_3} \tag{7.5}$$



Phase-Shifting Analysis: Unwrapping

This wrapped phase map includes the modulo 2π discontinuity, as shown in Fig 7.8.
The continuous phase map Φ(i, j) can be obtained by use of a phase-unwrapping algorithm as shown in Fig 7.9.

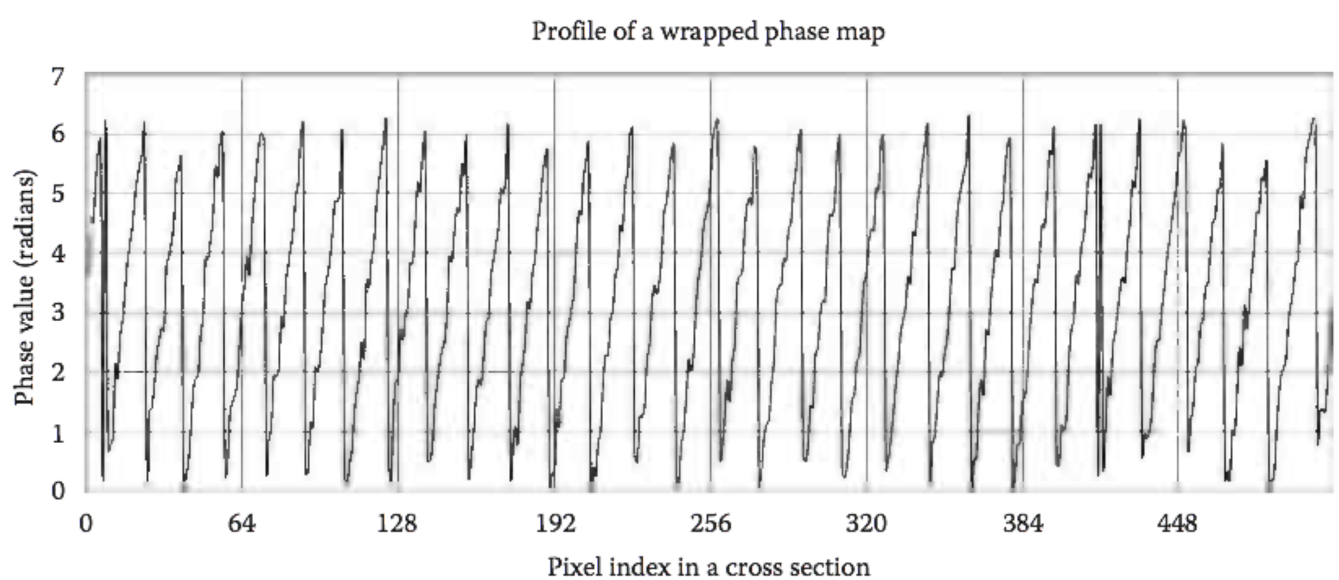


FIGURE 7.8
Profile of a wrapped phase map with 2π discontinuity.

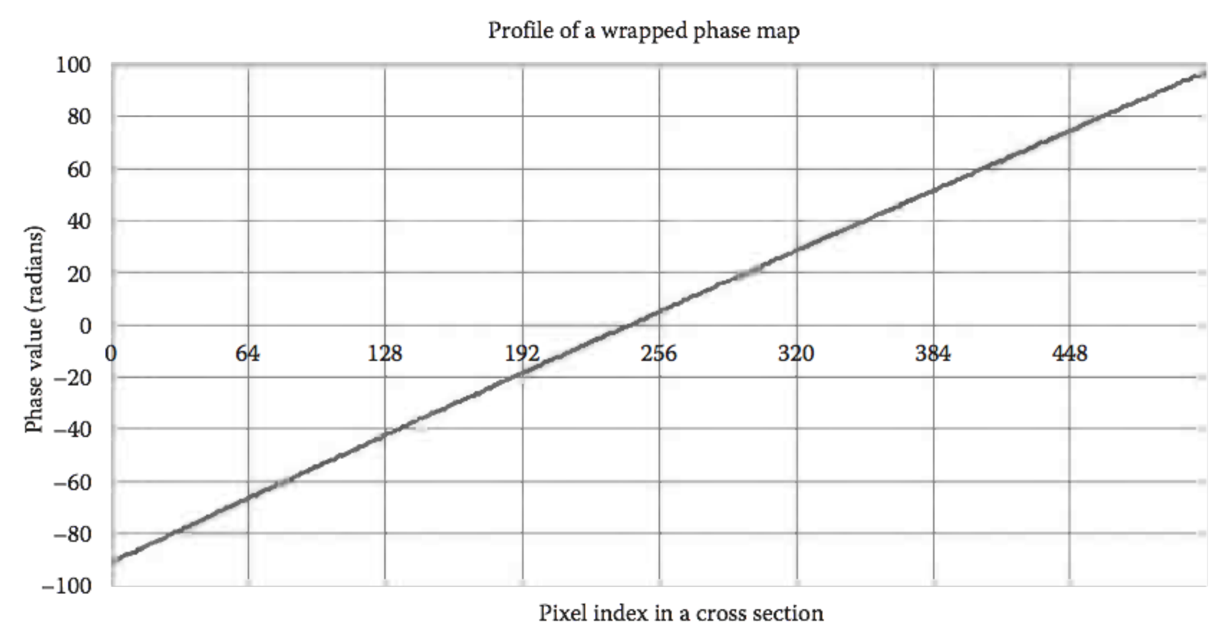


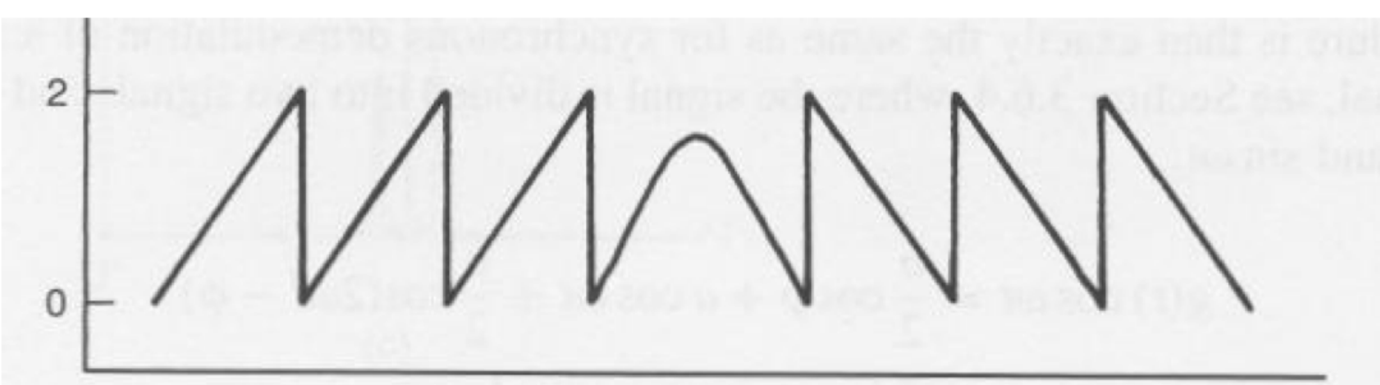
FIGURE 7.9
Profile of an unwrapped phase map without 2π discontinuity.

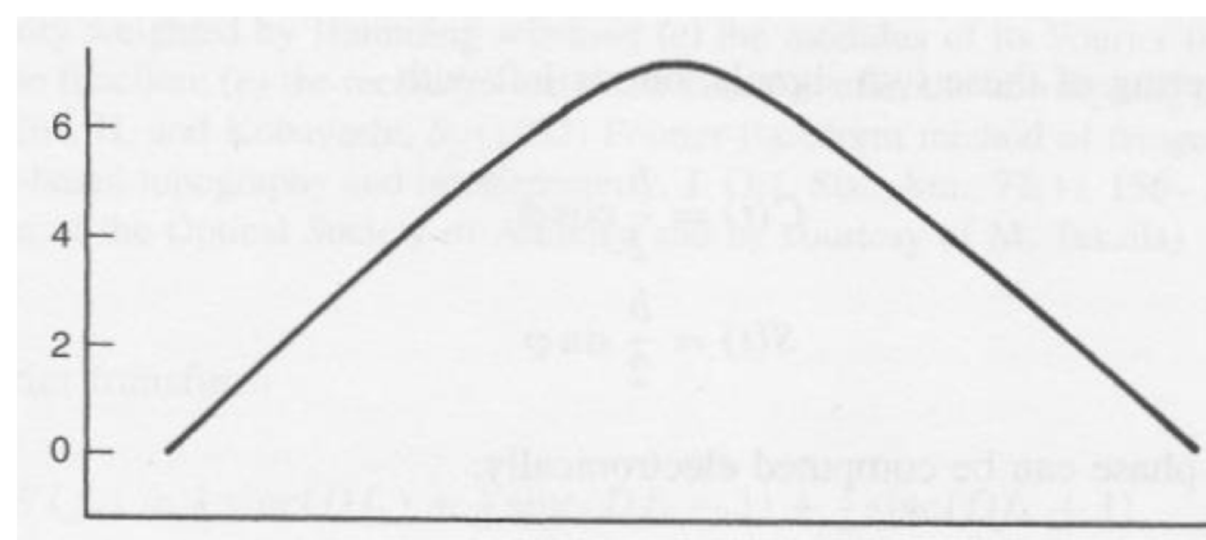
Phase-Shifting Analysis: Unwrapping

Discountinuties occur if Φ changes by 2π

If Φ is increasing, the slope is positive and vice versa

Final step in the fringe analysis is to unwrap or integrate the phase along a line (or path) counting the discontinuities and ± 2π each time the phase angle jumps from 2π to 0 or 0 to 2π





(b)

Benefits of Phase-Shifting Analysis

- Phase-shifting analysis enables full-field analysis in areas.
- The obtained phase map provides directional information.
- Phase-shifting analysis obtains phase information from image contrast, not intensity changes from peak to valley, thus enabling much higher accuracy and making the analysis tolerant to various surface finishes.

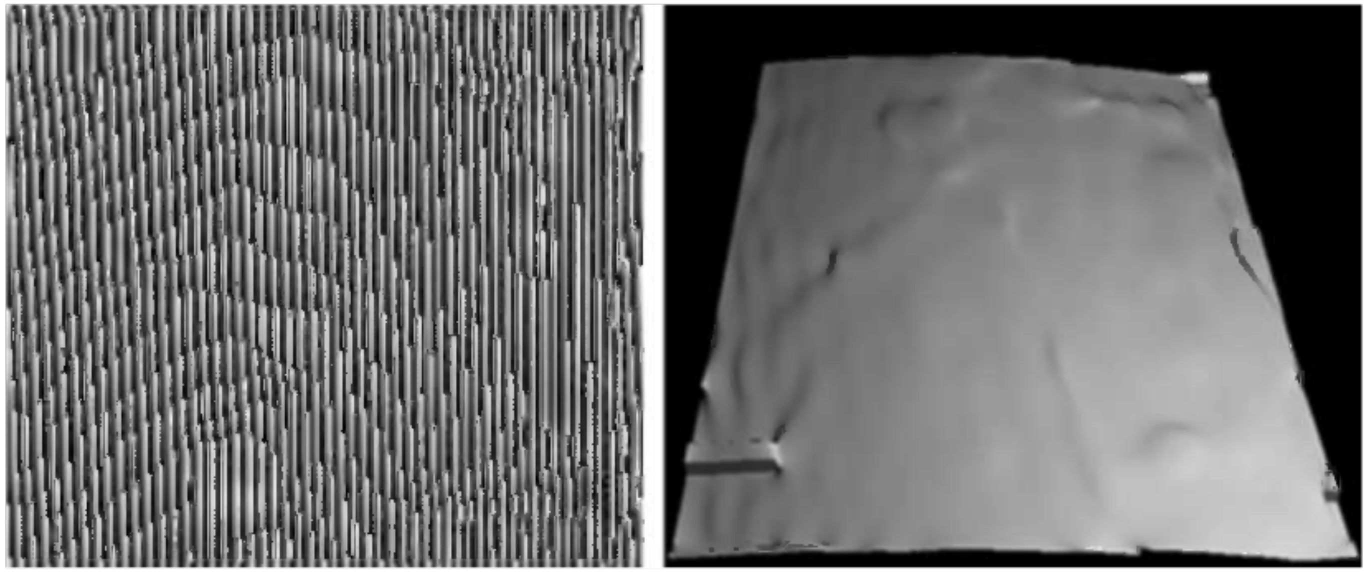

(a) (b)

FIGURE 7.10
Wrapped phase map (a) and unwrapped phase map (b) after bringing down.

Phase-Shifting Systems

Physical Phase-Shifting System.

- A translation stage is used to translate a component, e.g. in a Michelson interferometer a mirror is translated to introduce phase shifting. The relation between phase shift ø and translation offset δ (λ is the wavelength of light source):

$$\varnothing = 4\pi \frac{\delta}{\lambda}$$

- In a projection moiré the grating is translated and translation offset δ (p is the pitch of physical grating):

$$\varnothing = 2\pi \frac{\delta}{p}$$

Phase-Shifting Systems

Digital Phase-Shifting System.

A digital projector is used to project software-generated fringe patterns.

For a straight-line sinusoidal fringe pattern, the equation used to generate the fringe image in the computer can be written as:

$$I(u,v) = \frac{M}{2}\left[1+\cos\left(2\pi\frac{u}{p}+\theta\right)\right] \tag{7.8}$$

where

$I(u, v)$ is the gray level at point (u, v) in the projector chip (LCD, DMD, or LCOS)
p is the period of the fringe pattern in pixels
M is the maximum grayscale the project supports
θ is the phase shift

The fringe line is along the v direction.

Phase-Shifting Algorithms for Phase Wrapping

General Phase-Shifting Algorithm.

The captured 2D fringe image can be written as:

$$I_k(i,j) = I_0(i,j)\left[1+\gamma(i,j)\cos\left(\phi(i,j)+\theta_k\right)\right], \quad k=1,2,3,\ldots,K \tag{7.11}$$

or

$$I_k(i,j) = I_0(i,j) + I'(i,j)\cos\left(\phi(i,j)+\theta_k\right), \quad k=1,2,3,\ldots,K \tag{7.12}$$

where
- k is the index number of the images used in the phase measurement method
- I_k is the intensity at pixel (i, j) in the captured image
- I_0 is the background illumination
- γ is the fringe modulation (representing image contrast)
- I' is the image contrast
- θ_k is the initial phase for the kth image
- K is the total number of the fringe images

Phase-Shifting Algorithms for Phase Wrapping

Three-Step Phase-Shifting Algorithm.

In the three-step phase-shifting algorithm,[17,38] the phase shift $\theta = -2\pi/3$, 0, and $2\pi/3$ is used for three fringe images, respectively. The intensities of the three phase-shifted images at pixel (i, j) are

$$I_1(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j) - \frac{2\pi}{3}\right] \tag{7.13}$$

$$I_2(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j)\right] \tag{7.14}$$

$$I_3(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j) + \frac{2\pi}{3}\right] \tag{7.15}$$

Phase-Shifting Algorithms for Phase Wrapping

Four-Step Phase-Shifting Algorithm.

The four-step phase-shifting algorithm uses four fringe images with shifted phase θ as

$$\theta_i = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \quad i = 1, 2, 3, 4 \tag{7.22}$$

The four images can be written as

$$I_1(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j)\right] \tag{7.23}$$

$$I_2(i,j) = I(i,j) + I'(i,j)\cos\left[\varphi(i,j) + \frac{\pi}{2}\right] \tag{7.24}$$

$$I_3(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j) + \pi\right] \tag{7.25}$$

$$I_4(i,j) = I(i,j) + I'(i,j)\cos\left[\varphi(i,j) + \frac{3\pi}{2}\right] \tag{7.26}$$

Using these trigonometric functions, the phase information can be calculated as

$$\phi(i,j) = \tan^{-1}\left(\frac{I_4 - I_2}{I_1 - I_3}\right) \tag{7.27}$$

Phase-Shifting Algorithms for Phase Wrapping

Carré Phase-Shifting Algorithm.

The Carré phase-shifting algorithm is a four-step phase shifting algorithm for use with an unknown phase shift. The four images can be written as

$$I_1(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j) - 3\theta\right] \tag{7.29}$$

$$I_2(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j) - \theta\right] \tag{7.30}$$

$$I_3(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j) + \theta\right] \tag{7.31}$$

$$I_4(i,j) = I(i,j) + I'(i,j)\cos\left[\phi(i,j) + 3\theta\right] \tag{7.32}$$

In this four-equation group, there are four unknowns. The phase ϕ can be calculated as

$$\phi(i,j) = \tan^{-1}\left(\frac{\sqrt{3(I_2 - I_3)^2 - (I_1 - I_4)^2 + 2(I_2 - I_3)(I_1 - I_4)}}{(I_2 + I_3) - (I_1 + I_4)}\right) \tag{7.33}$$

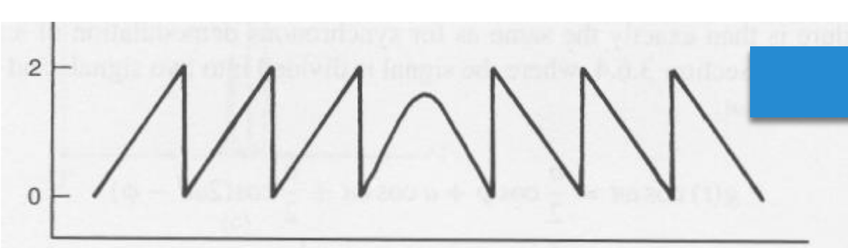
Phase-Shifting Algorithms for Phase Wrapping

Selection of Phase-Shifting Algorithms.

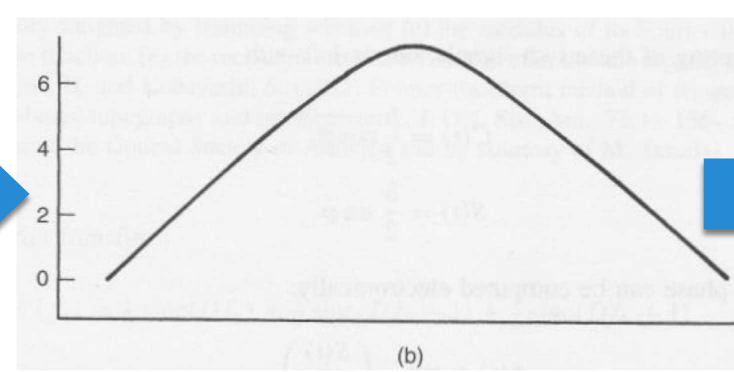
- Many different phase-shifting algorithms have been developed in the past that can be used to reduce different types of errors.
- For digital phase-shifting systems, phase shift is performed by software programming, and there is no phase-shifting error. The main error sources become the nonlinearity and noise from the camera and projector. Those phase-shifting algorithms that are insensitive to the system nonlinearity will provide the best measurement results.
- For physical phase shifting, the phase-shifting error is usually one of dominant error sources. When using a physical phase-shifting method, the algorithms that are insensitive to the phase-shift error will work best.

Phase-Shifting System Modeling and Calibration

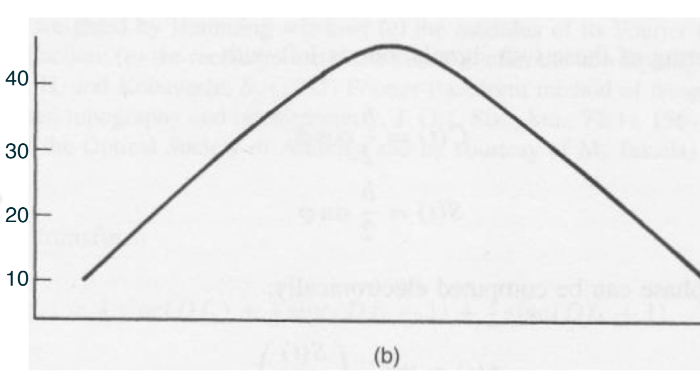
- Phase-shifting algorithms for phase wrapping result in a phase map with a 2π ambiguity. To remove the it, a phase-unwrapping process is needed.
- There is not a unique unwrapped phase maps $\phi(i, j)$.
- To covert from $\phi(i, j)$ to coordinates of the surface points requires a unique absolute phase map, system modeling, and system calibration.



Wrapped phase (rad)



Unwrapped phase (rad)



Profile (microns)

Unwrapped Phase Map and Absolute Phase Map

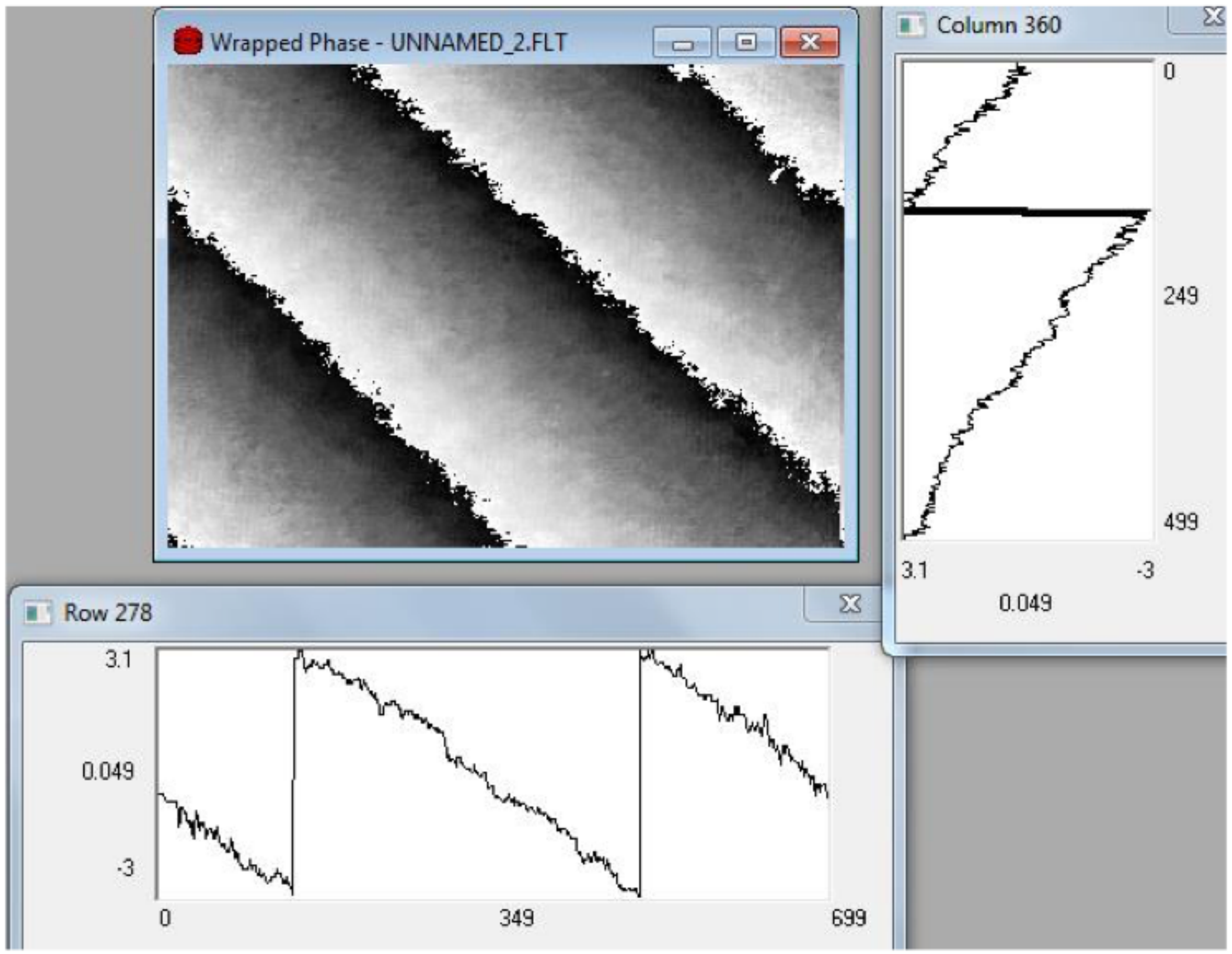
- Unwrapped phase map is a continuous phase map without 2π discontinuity.

- Absolute phase map is the phase map to be converted to coordinates.

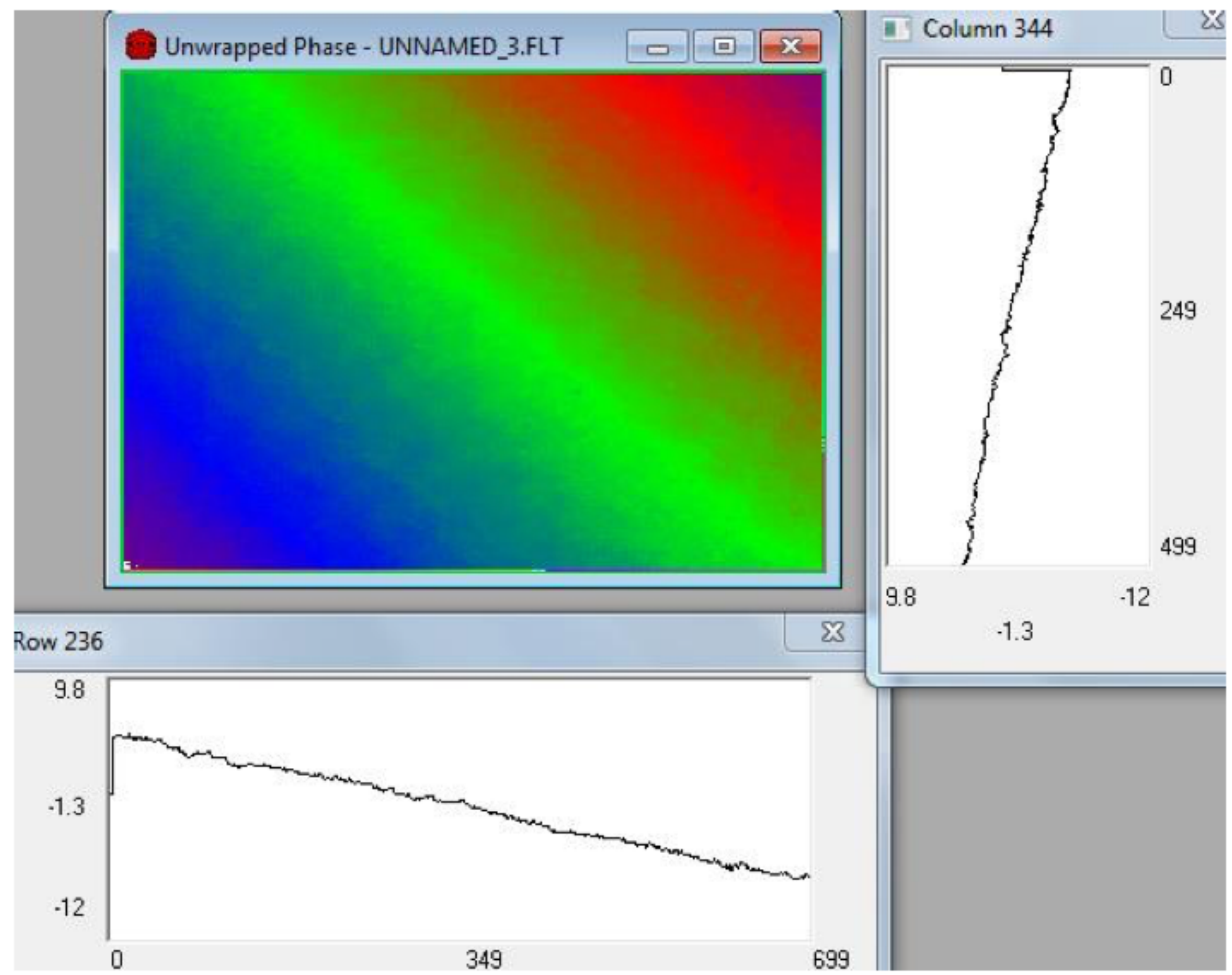
- For a linear or partially linear model, the absolute phase map is usually obtained by subtracting a reference phase map from the measurement phase map.

Fringe Analysis - Phase

$$Tan\delta = \frac{I_4 - I_2}{I_1 - I_3}$$

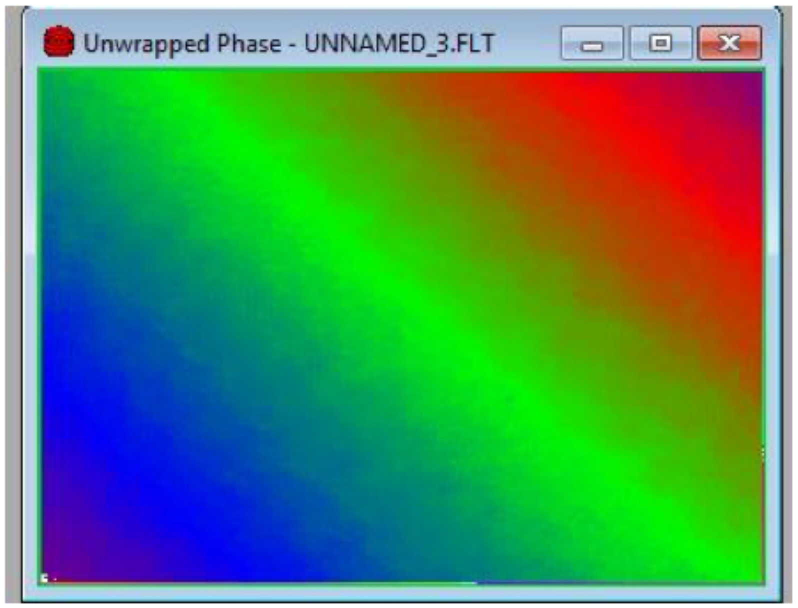


- Since the image is tan δ the values at between ±π hence the saw tooth image

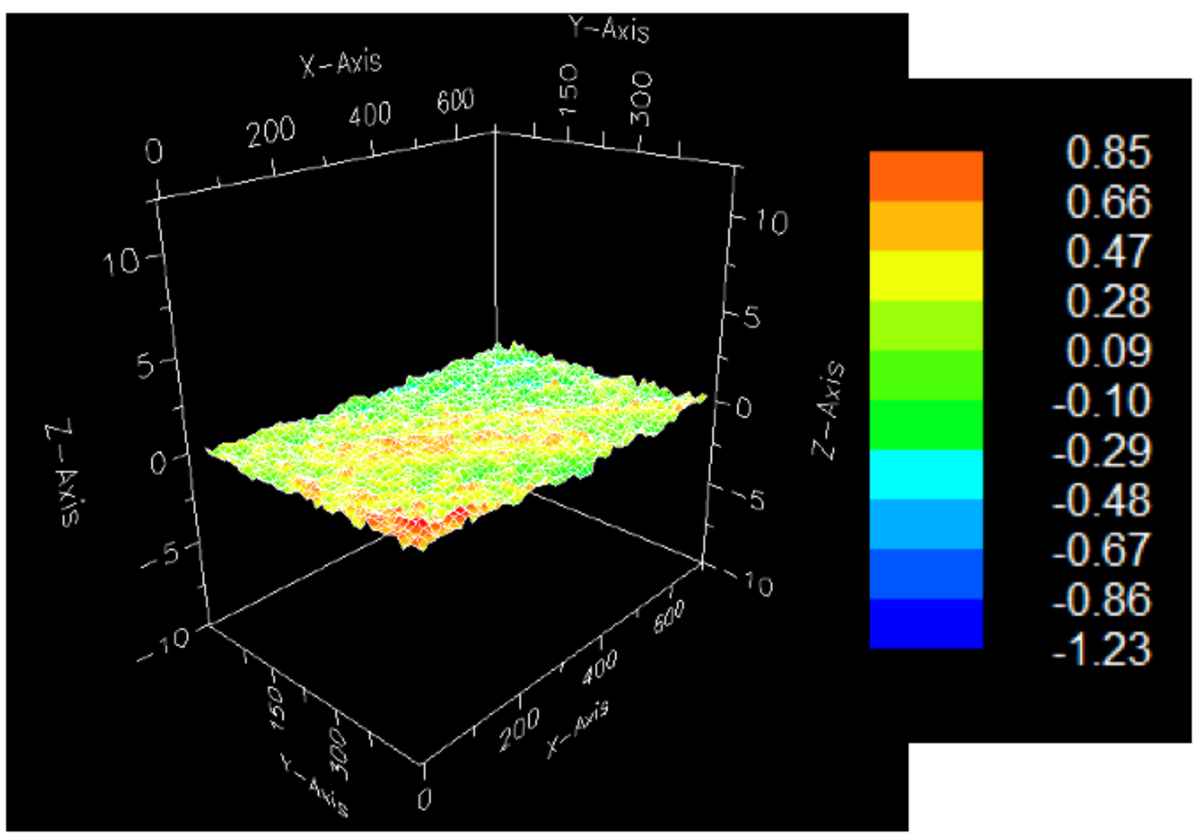


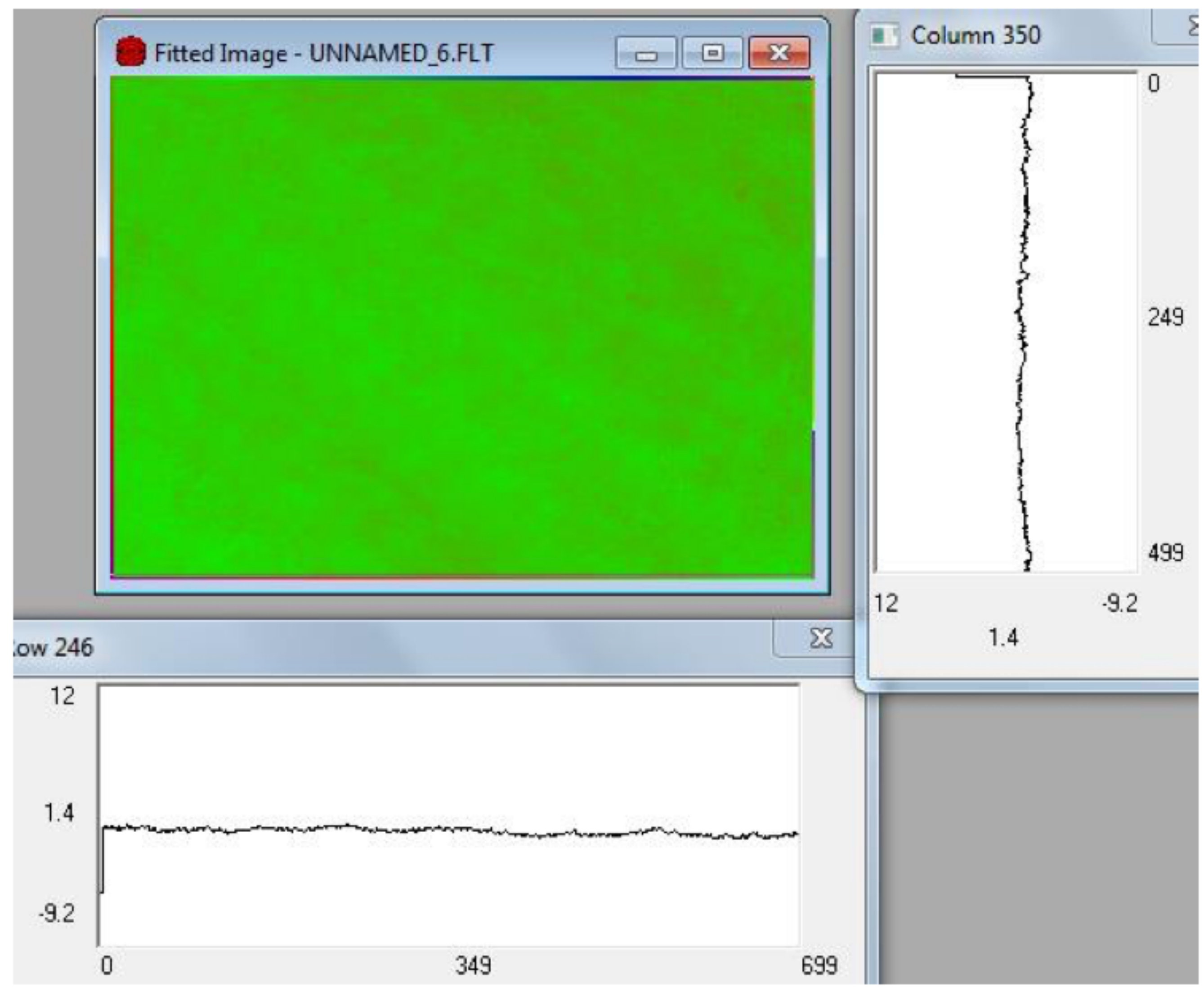
- Unwrapping provides the continuous phase image

Fringe Analysis - Phase



- Since the tilt is constant to get the fringes, it can be removed by least square fitting.

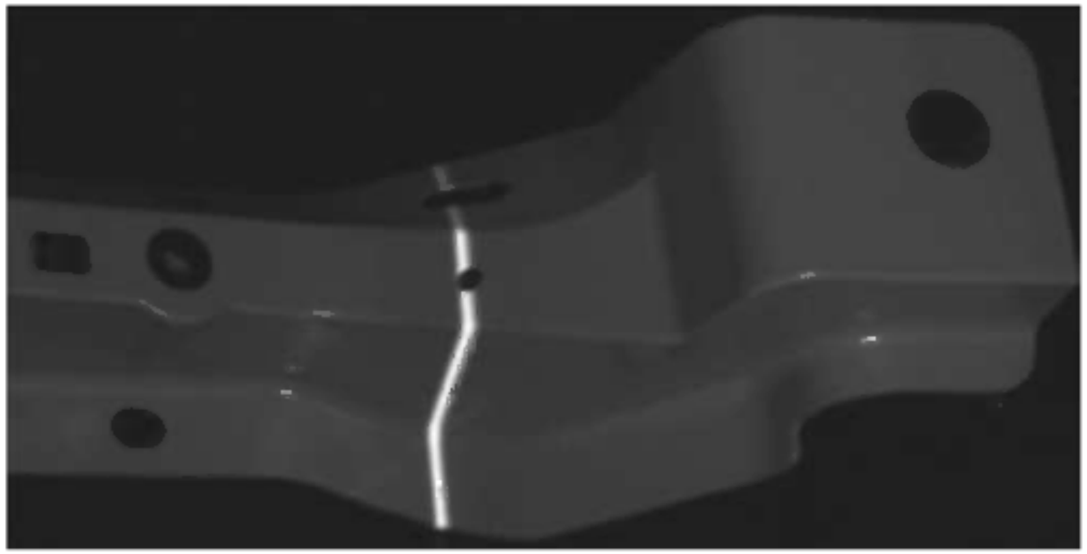




Unwrapped Phase Map and Absolute Phase Map

Reference phase maps can be obtained by either performing phase shifting on a reference plane (usually flat) or by creating a flat phase plane determined by specific points (constituting a horizontal and a vertical line) on the measurement phase map.

The coordinates calculated later will be referred to these planes. After subtraction, the measurement phase map is brought down.



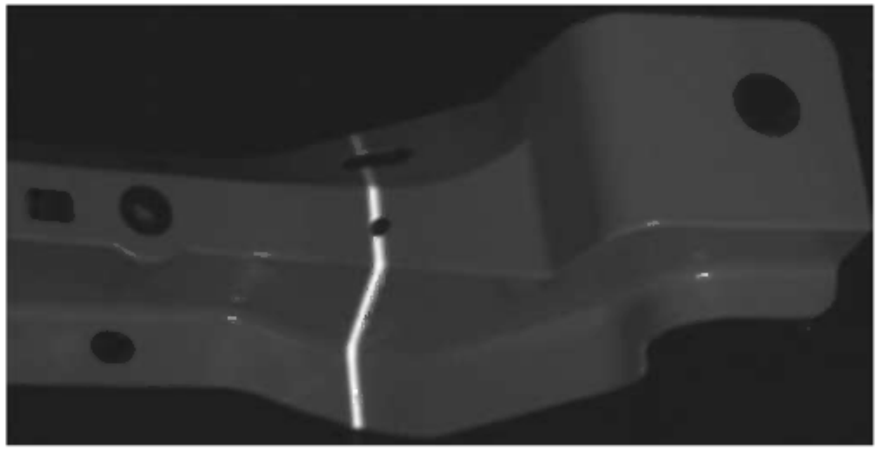
Unwrapped Phase Map and Absolute Phase Map

The centerline image was used to identify the pixels in the phase map that correspond to the centerline in the projector chip. These pixels should have the same absolute phase as that of the centerline of the projection field where the project fringe patterns were programmed. With the absolute phase at these pixels known, the absolute phase map of the entire surface can be obtained by simply translating the relative unwrapped phase map $\Phi(i, j)$. Assume the absolute phase of the centerline to be Φ_0. The absolute phase map $\Phi'(i, j)$ can be obtained as follows:[38]

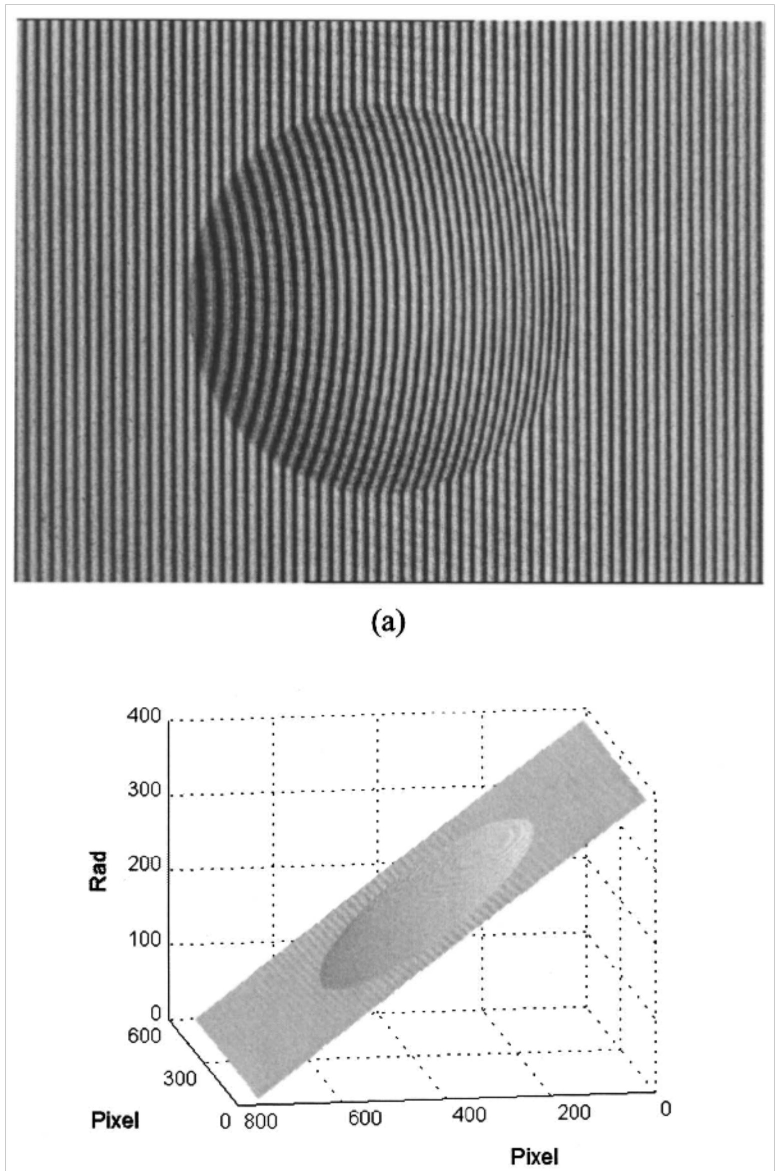
$$\Phi'(i, j) = \Phi(i, j) + \Phi_0 - \frac{1}{N}\sum_{k=1}^{N}\Phi_k \tag{7.43}$$

where

Φ_k's are the phases of the pixels that correspond to the centerline of the projection field
N is the total number of such pixels in a specified segment

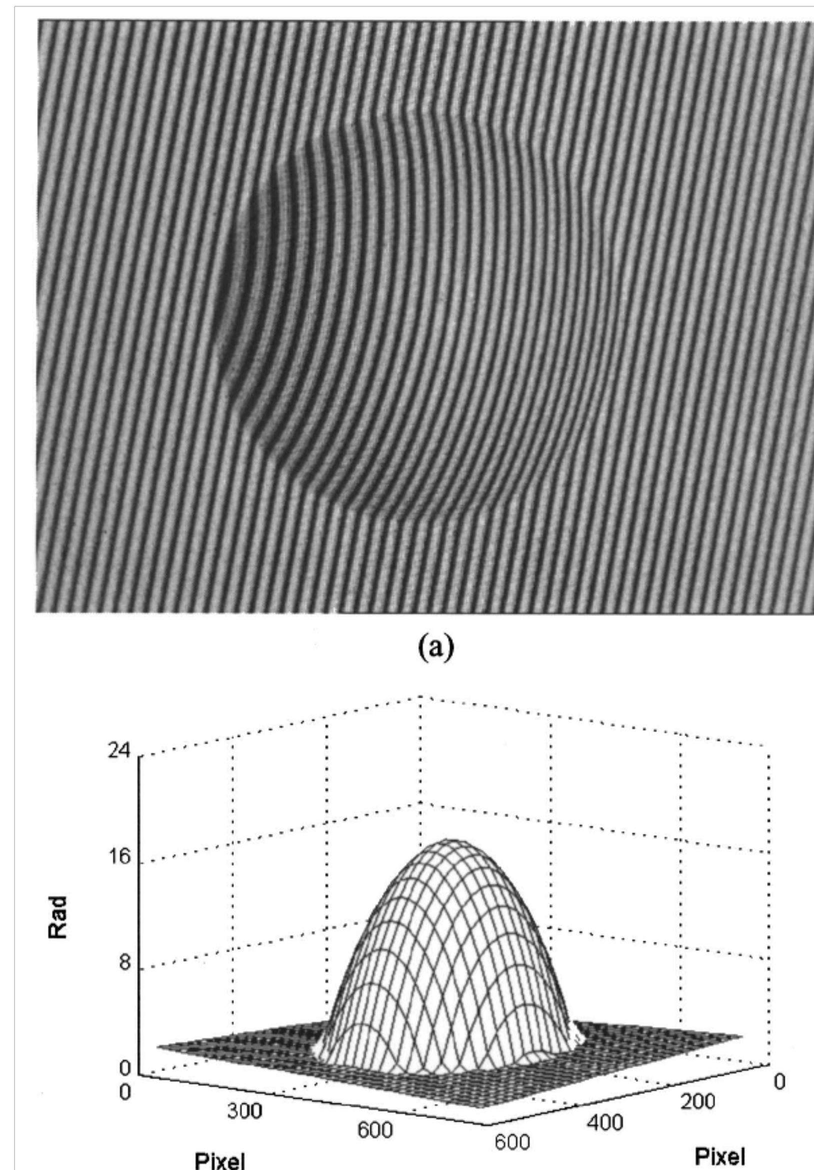


Unwrapped Phase Map and Absolute Phase Map



(a)

(b)



(a)

Linear Model for Flat Surface Measurement

As a simple model that many researchers like to use, the linear model is very straightforward: the lateral dimensions are proportional to the pixel index while the vertical dimension is proportional to the absolute phase after reference phase subtraction. The calculation that converts pixel (i, j) with absolute phase Φ' to coordinates (x, y, z) can be represented by the following formulae:[53,54]

$$x = K_x(i - C_x) \tag{7.44}$$

$$y = K_y(j - C_y) \tag{7.45}$$

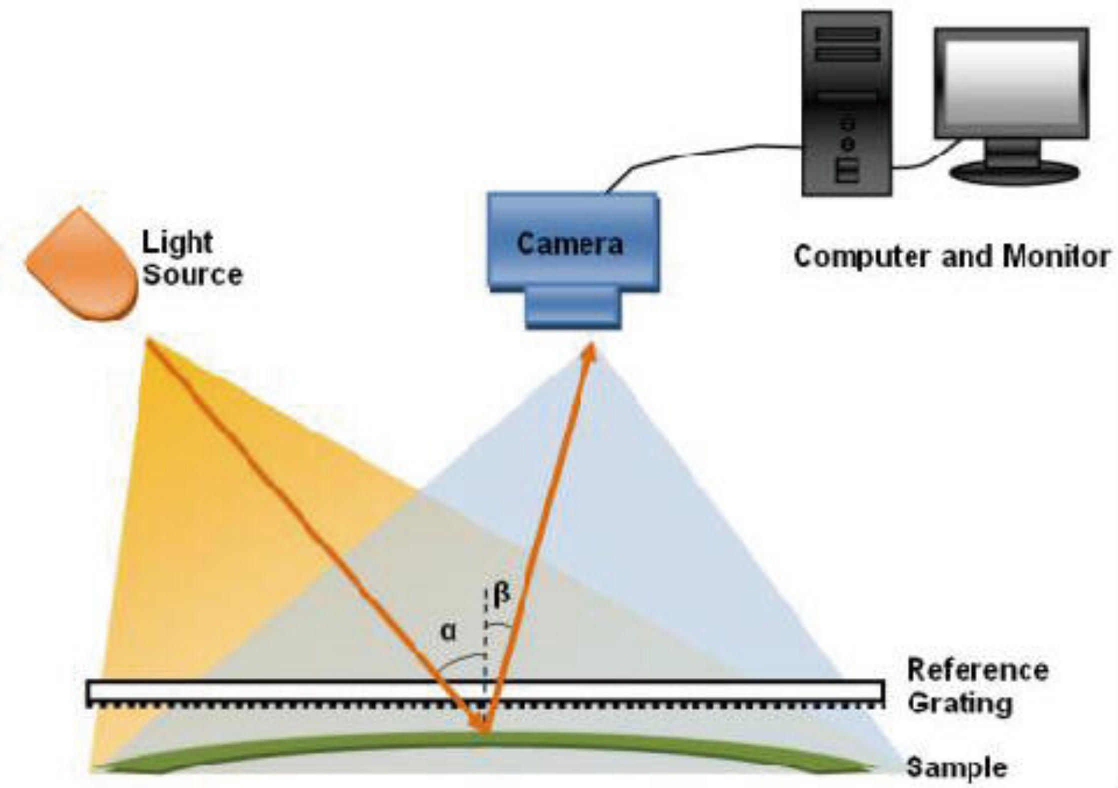
$$z = k_z\Phi' \tag{7.46}$$

where

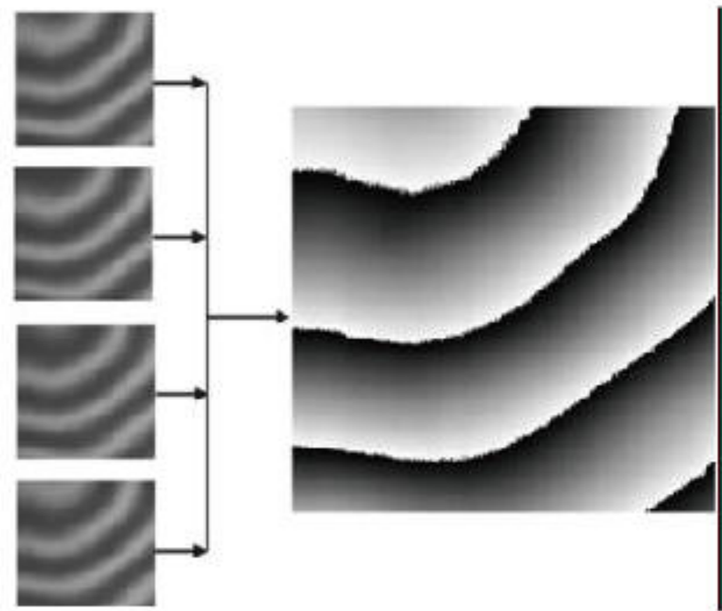
K_x, K_y, K_z are scalars in the three coordinate directions
(C_x, C_y) are specified coordinate origin in the lateral directions

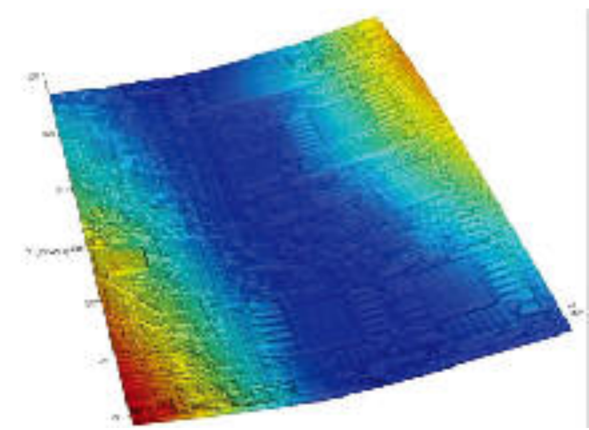
In practice, K_x and K_y are usually determined by calibration in the FOV and K_z is determined by step gage standards.

Linear Model for Flat Surface Measurement



Shadow Moiré System Diagram


Combining Four Intensity Images into One Phase Image



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Partially Linear Model for Flat Surface Measurement

The partially linear model assumes that some dimensions (usually x and y coordinates) are proportional to the pixel index (i, j) on the image while the vertical coordinate is calculated from the absolute phase value using a nonlinear formula.

To deduce this kind of functions, some assumptions must be made such as assuming the camera is at the same height as the grating and/or assuming the optical axis of the camera/lens is perpendicular to the object surface. Using this method, the system configuration of a fringe projection system can be simplified as shown in Figure 7.13. Let (C_x, C_y) be the intersect of the camera sensing surface and the optical axis of the imaging lens; the coordinates (x, y, z) can be calculated as

$$x = K_x(i - C_x) \tag{7.47}$$

$$y = K_y(j - C_y) \tag{7.48}$$

$$z = h = \frac{L \times \Phi'}{\Phi' - 2\pi f D} \tag{7.49}$$

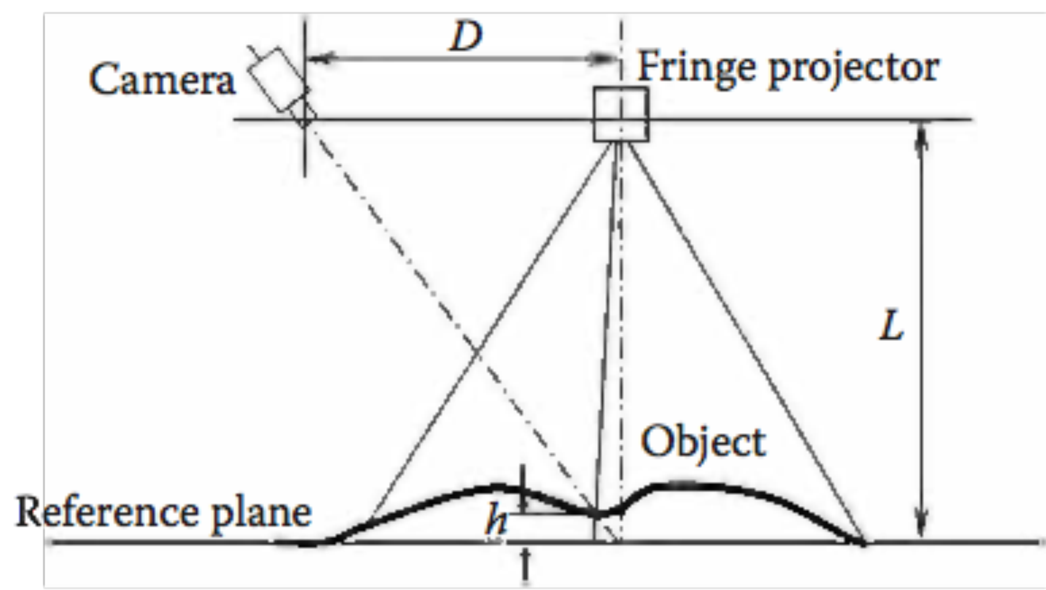
where K_x, K_y are scalars in the lateral directions determined by calibrating the FOV. In Equation 7.49, Φ' is the absolute phase, that is, phase difference at pixel (i, j) between the flat reference plane and the object plane, and f is the average frequency of fringe on reference plane. Φ' can be obtained by either subtracting the object phase map from the phase map

Partially Linear Model for Flat Surface Measurement

$$x = K_x(i - C_x)$$

$$y = K_y(j - C_y)$$

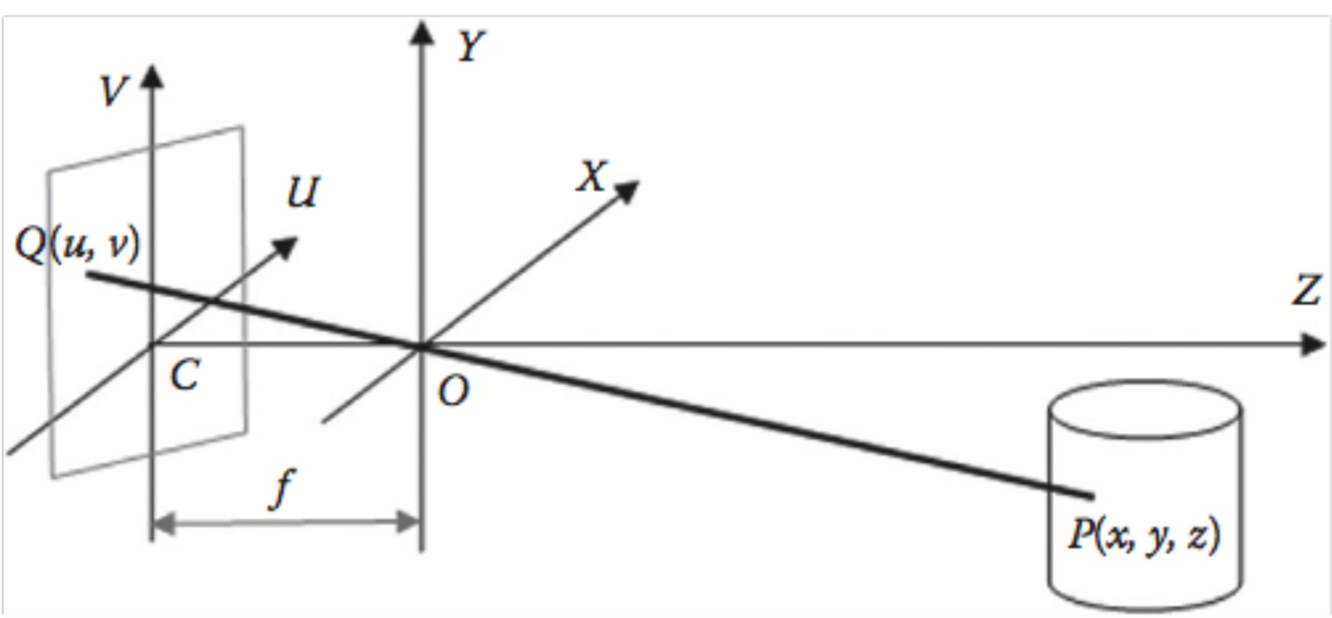
$$z = h = \frac{L \times \Phi'}{\Phi' - 2\pi f D}$$



Structured light system calibration

Camera Calibration.

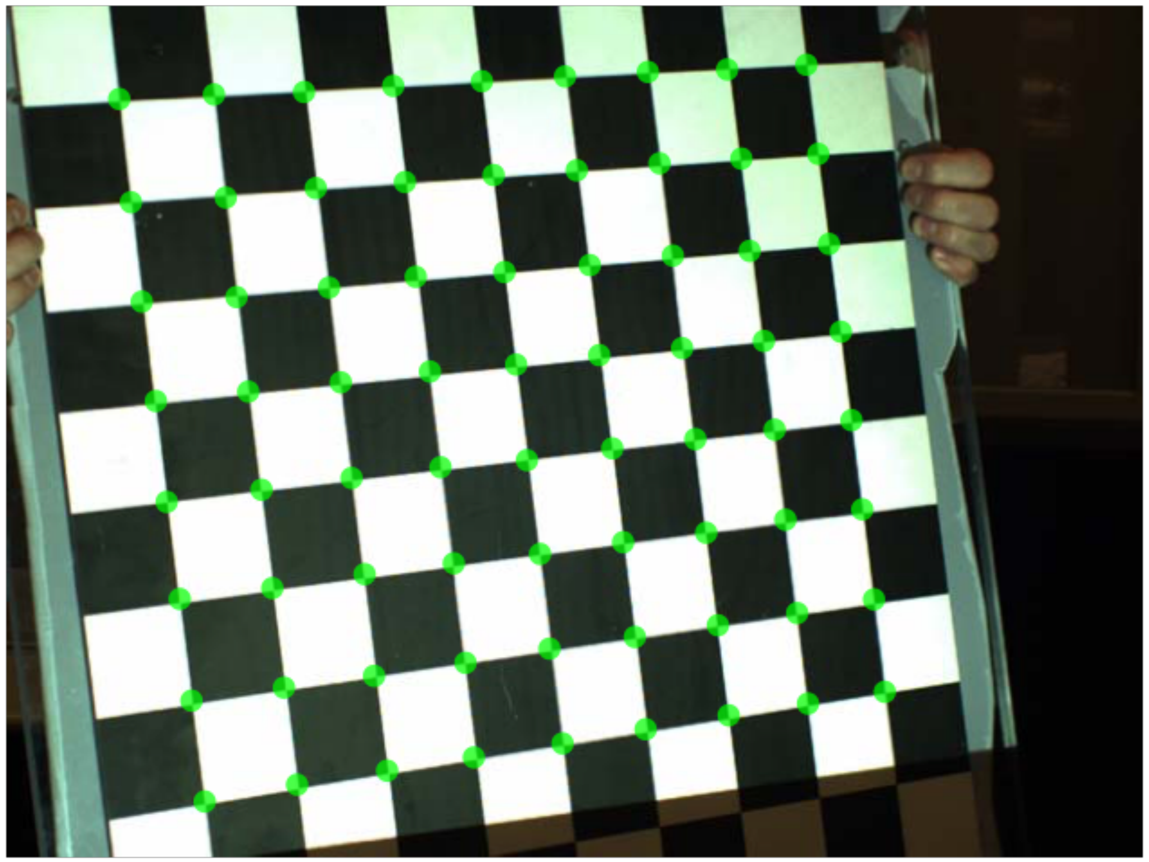
A camera model describes the mapping between points in a 3D space and a pixel in the 2D camera sensor chip. The parameters in a camera model can be classified into intrinsic parameters, which describe the geometry of the camera itself, and extrinsic parameters, which determine the camera's pose in the 3D space.

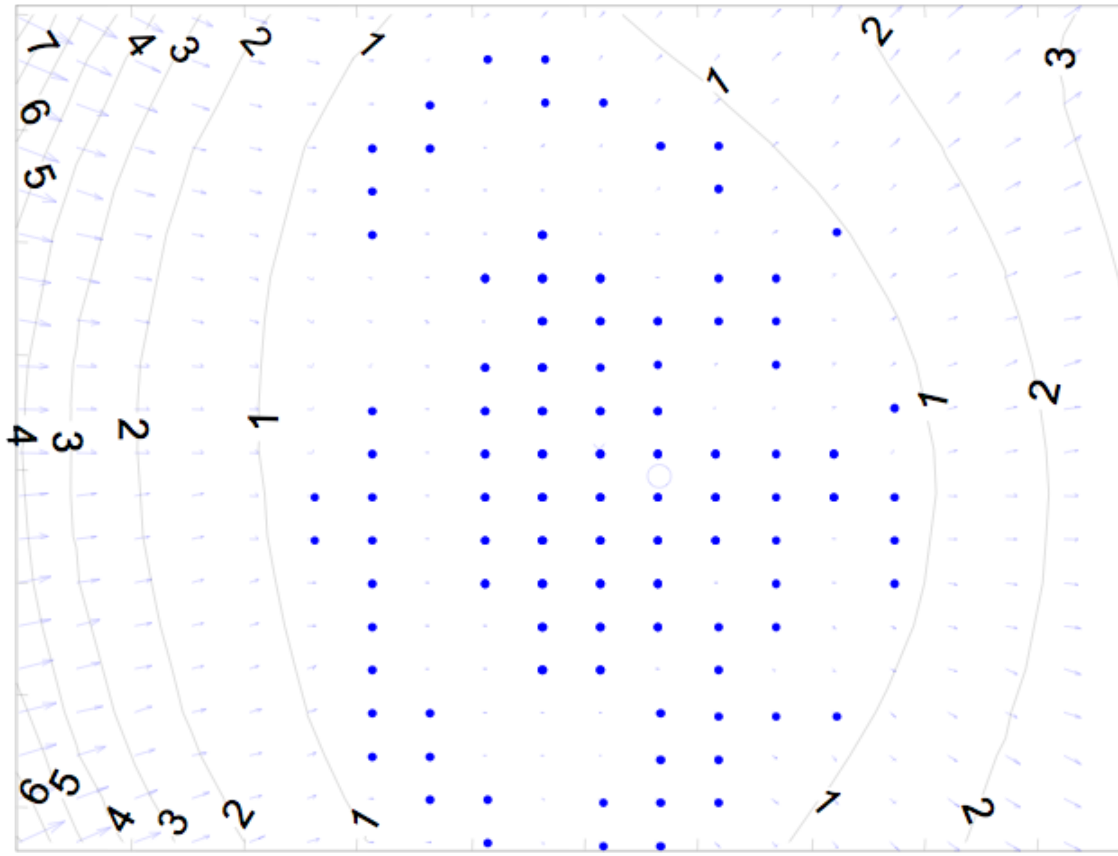


$$u = \frac{fx}{z}$$

$$v = \frac{fy}{z}$$

Overview of Projector-Camera Calibration





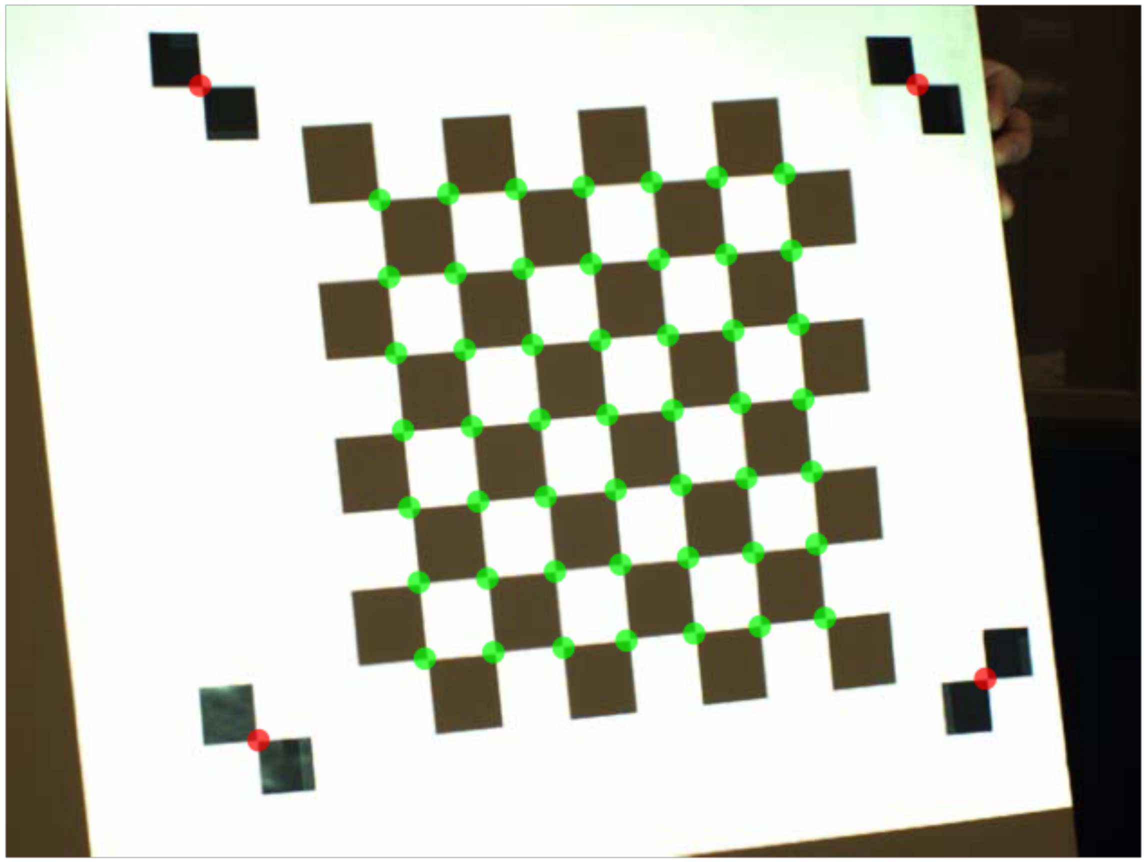
Estimated Camera Lens Distortion

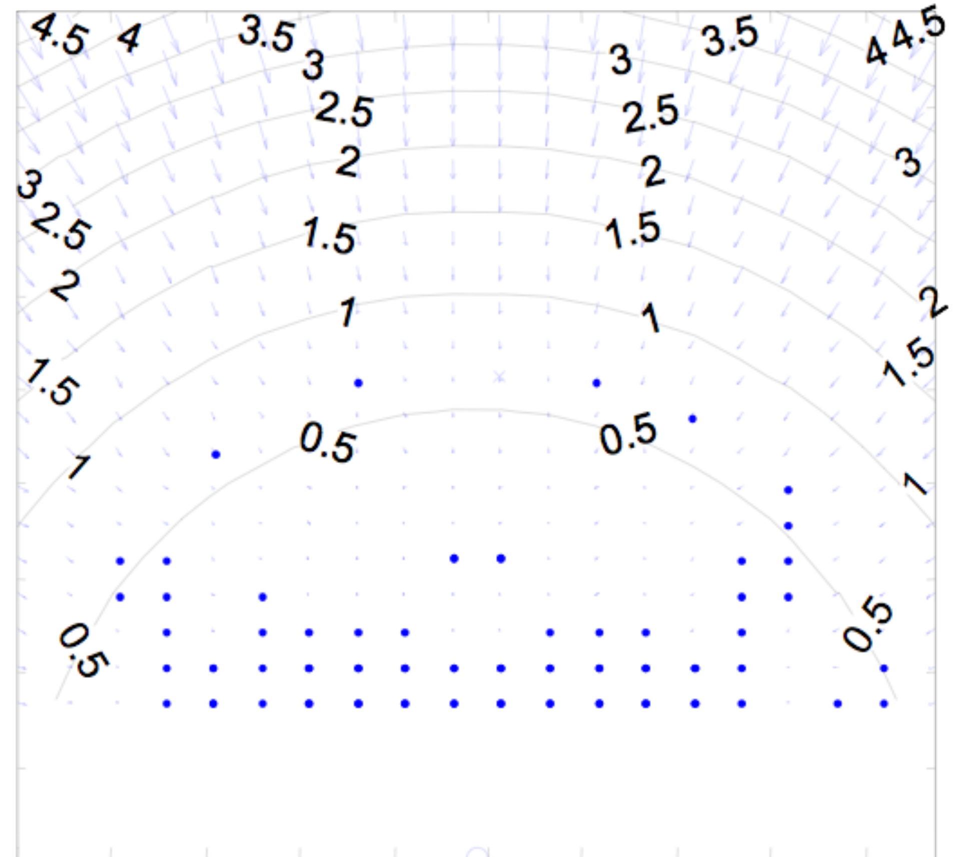
Camera Calibration Procedure

- Uses the *Camera Calibration Toolbox for Matlab* by J.-Y. Bouguet

Normalized Ray	Distorted Ray (4th-order radial + tangential)	Predicted Image-plane Projection
$x_n = \begin{bmatrix} X_c/Z_c \\ Y_c/Z_c \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$	$x_d = \begin{bmatrix} x_d(1) \\ x_d(2) \end{bmatrix} = \left(1 + kc(1)\, r^2 + kc(2)\, r^4 + kc(5)\, r^6\right) x_n + dx$ $dx = \begin{bmatrix} 2\, kc(3)\, x\, y + kc(4)\left(r^2 + 2x^2\right) \\ kc(3)\left(r^2 + 2y^2\right) + 2\, kc(4)\, x\, y \end{bmatrix}$	$x_p = fc(1)\left(x_d(1) + alpha\_c * x_d(2)\right) + cc(1)$ $y_p = fc(2)\, x_d(2) + cc(2)$

Overview of Projector-Camera Calibration



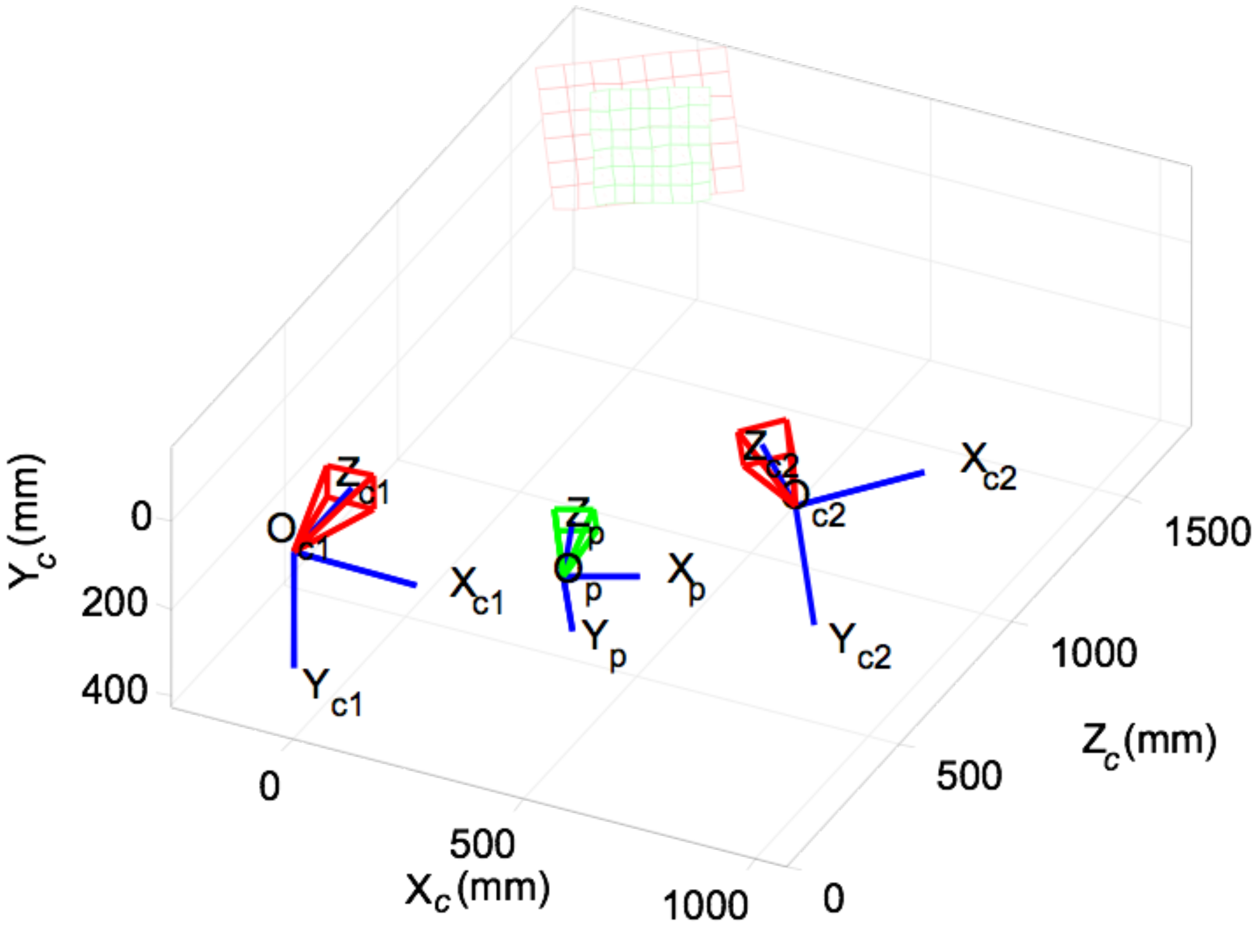


Estimated Projector Lens Distortion

Projector Calibration Procedure

- Consider projector as an inverse camera (i.e., maps intensities to 3D rays)
- Observe a calibration board with a set of fidicials in known locations
- Use fidicials to recover calibration plane in camera coordinate system
- Project a checkerboard on calibration board and detect corners
- Apply ray-plane intersection to recover 3D position for each projected corner
- Use *Camera Calibration Toolbox* to recover intrinsic/extrinsic projector calibration using 2D→3D correspondences with 4th-order radial distortion

Overview of Projector-Camera Calibration



Projector-Camera Calibration Results

- Implemented complete toolbox for projector-camera calibration
- Sufficient accuracy for structured lighting applications
- Future version will incorporate final global bundle adjustment